A game-theoretic approach to parallel trade through the price of anarchy

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Abstract

In recent years, the concept of price of anarchy has emerged as a tool to measure the efficiency of Nash equilibria in noncooperative games. In this paper, we apply an adaptation of this concept to subgame-perfect Nash equilibria arising in three different dynamic noncooperative game-theoretic models for the parallel trade of pharmaceuticals, where parallel trade refers to the arbitrage opportunity created when the same drug is sold in two countries, with differences in prices and/or reimbursement regulations. More specifically, for three different expressions of the global welfare of two countries, we find in closed form its optimal value, then we evaluate for such expressions the prices of anarchy associated with the subgame-perfect Nash equilibria of three dynamic noncooperative games modeling the interaction between a manufacturer in the first country and a distributor in the second country, respectively in the case of parallel trade banning for the distributor, parallel trade threat from the distributor (but no occurrence of parallel trade at equilibrium), and actual parallel trade occurrence at equilibrium. Finally, we evaluate the dependence of such prices of anarchy on the relative market size of the exporting country with respect to the importing one, and on the per-unit parallel trade cost. Extensions of the methodology to other noncooperative game-theoretic models of parallel trade of pharmaceuticals are discussed.

Keywords:
Parallel trade freedom; pharmaceuticals; noncooperative game theory;

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subgame-perfect Nash equilibrium; price of anarchy.

1. Introduction

In general, drug prices vary across countries. Differences in pricing and reimbursement regulations may create arbitrage opportunities, which is known as parallel trade. When parallel trade is allowed by policymakers, parallel traders may buy drugs in a country where prices are lower, then re-sell them in a country where prices are higher. As locally sourced and parallel traded drugs are produced by the same manufacturer, they are exactly the same (apart from possibly different packagings), which creates opportunities for potential parallel traders. According to some policymakers, parallel trade should be permitted as a means to reduce pharmaceutical prices in re-importing countries. However, manufacturers often see parallel trade as a potential threat to their investments in research and development. As a consequence, there is a strong debate about the opportunity or not of permitting parallel trade [1, 2, 3].

Since parallel traders and manufacturers are different agents with their own objectives, parallel trade and its consequences on the global welfare of the involved countries have been investigated in the literature through various noncooperative game-theoretic models [2, 4, 5, 6, 7]. For instance, according to the model developed in [5], restricting parallel trade is always advantageous for the manufacturer, but it may either increase or decrease the global welfare of the two countries. However, a different model is used in [2], showing that parallel trade may even increase the profit of a pharmaceutical firm, depending on its bargaining power and on the relative market size of the exporting country with respect to the importing one. In [4], two dynamic noncooperative games are proposed to investigate the equilibrium behavior of a manufacturer located in a country and a distributor belonging to a second country, when parallel trade from the distributor is, respectively, permitted/forbidden. In the first case, it is shown therein that parallel trade actually does not even occur at equilibrium (i.e., the quantity of re-imported product from the parallel trader is 0), but the threat of potential parallel trade (or parallel trade freedom) influences the equilibrium behavior of both players, changing the equilibrium prices and quantities of the product sold by each of them. In [6], parallel trade is examined through infinitely-repeated noncooperative games with perfect and complete information, investigating the effect of different policies enabling or not parallel trade on the associated subgame-perfect Nash equilibria. Differently from [4], in this case parallel trade actually occurs at equilibrium, when parallel trade is permitted. This also happens for the above-mentioned model investigated in [5]. Finally, for other noncooperative game-theoretic models of parallel trade, we refer the interested reader to the monograph [7].

Within this noncooperative game-theoretic framework, in the paper we propose the use of the concept of price of anarchy[1] [9], as a means to investigate the efficiency of solutions to noncooperative game-theoretic models of parallel trade. In the present context, the price of anarchy is the ratio between the optimal value of the global welfare (i.e., the one obtained by a hypothetical global planner, by solving a suitable optimization problem) and its value obtained in correspondence of the "worst" equilibrium of the game. More specifically, in the paper we evaluate the price of anarchy for the two above-mentioned dynamic noncooperative games proposed in [7] to model the interaction between a manufacturer and a distributor, assuming, respectively, parallel trade banning for the distributor and parallel trade threat from the distributor (but no occurrence of parallel trade at equilibrium), and for the dynamic noncooperative game proposed in [5], for which there is an actual occurrence of parallel trade at equilibrium. Hence, we evaluate the effect of different levels of parallel trade freedom on the price of anarchy. In order to compute the latter, we consider three different models for the global welfare function of the two countries, i.e., the Bentham model and two specifications of the Rawls model. Then, we compare the expressions of the price of anarchy obtained for the noncooperative games and global welfare models examined in the paper. The original contributions of the paper are, for each of the three models of the global welfare function, the evaluation of its optimal value in closed form, and the consequent computation of the prices of anarchy for the two noncooperative games proposed in [7] and the one proposed in [5].

The paper is organized as follows. Section 2 summarizes the two-country models for the trade of pharmaceuticals presented in [5] and [7]. In Section 3, we express in closed form the optimal value of the global welfare of the two countries for the Bentham and Rawls models, when parallel trade is permitted/forbidden. The two dynamic noncooperative games proposed in [7] to model parallel trade banning/threat are shortly summarized in Section 4, together with the one proposed in [5] to model the occurrence of parallel trade at equilibrium. Then, in Section 5, we evaluate for the Bentham and Rawls models the prices of anar-

[1] Originally proposed in [8] under the name of "coordination ratio"; nowadays, the term "price of anarchy" is more common [9].

chy associated with the subgame-perfect Nash equilibria of such games, and their dependence on the relative market size of the two countries and on the per-unit parallel trade cost. Finally, Section 6 discusses the obtained results and mentions possible extensions of the methodology to other noncooperative game-theoretic models of parallel trade investigated in the literature.

2. Background: a two-country model for the trade of pharmaceuticals

In this section, we summarize the models for the trade of pharmaceuticals considered in [5] and [7], involving two countries, characterized by different demand functions of one product possibly subject to parallel trade. The model includes both cases in which there is parallel trade freedom/banning. More precisely, the following is assumed in the model. The first country (named "country A" in the following) is the one in which a drug is fabricated by a manufacturer with a marginal cost of production equal to 0 (e.g., because the most relevant costs are the ones of research and development). Since the drug can be also sold in a second country (named "country B" in the following), the country A is the exporting country, whereas the country B is the importing country. However, when parallel trade from the country B to the country A is permitted, the country A is also the re-importing country. The demand functions of the drug in the two countries A and B are modeled by the following linear functions, respectively:

$$q_A = \gamma a - bp_A \,, \tag{1}$$

$$q_B = a - bp_B \,, \tag{2}$$

where q_A (respectively, q_B) is the quantity of the drug that the consumers in the country A (respectively, B) are willing to buy at the price p_A (respectively, p_B), $a, b > 0$ are two constants (the same for both countries), and $\gamma > 0$ is another constant, which describes the heterogeneity of the countries A and B with respect to the market size (indeed, in the limit case of very small prices, one gets $q_A \simeq \gamma a$ and $q_B \simeq a$, so in that case $q_A \simeq \gamma q_B$, and γ measures the relative market size of the country A with respect to the country B). After its production, the drug can be

(i) sold by the manufacturer of the country A to the consumers of the country A in quantity $q_A^{M,C}$ at the wholesale price $p_A^{M,C}$;

(ii) sold by the manufacturer of the country A to the distributor of the country B in quantity $q_B^{M,D}$ at the wholesale price $p_B^{M,D}$;

(iii) sold by the distributor of the country B to the consumers of the country B in quantity $q_B^{D,C}$ at the retail price $p_B^{D,C}$;

(iv) (only when there is parallel trade freedom) sold by the distributor of the country B to the consumers of the country A in quantity $q_A^{D,C}$ at the retail price $p_A^{D,C}$. In doing this, the distributor incurs a fixed per-unit parallel trade cost $t \geq 0$. When, instead, parallel trade is forbidden, one sets $q_A^{D,C} = 0$.

The model of trade considered in [5] differs from the one in [7] for the simplifying assumptions $\gamma = 1$ and $b = 1$ (which we do not make in the paper), and for the additional presence of a transfer payment $T_P \geq 0$ (franchise fee), which is paid by the distributor to the manufacturer, and which we also include in the model. Moreover, differently from [5] and [7], we also include in the model a total fixed cost of production $C_F \geq 0$, which can be interpreted as the cost of research and development, and is used later in the paper by the manufacturer to decide whether to do research and development (then, producing the drug), or not to do it (then, producing nothing). Finally, while the total fixed cost of production C_F cannot be modified, the transfer payment T_P can be set by to its maximum value for which the surplus of the distributor is non-negative, as done in [5]. Notice that, according to the model above, only the distributor can sell to the consumers in the country B. When there is parallel trade freedom, the distributor can also sell to the consumers in the country A (in the model, parallel trade from the consumers in the country B to the consumers in the country A is always forbidden). Of course, one has also the constraint $q_B^{M,D} = q_A^{D,C} + q_B^{D,C}$, i.e., the total quantity of the product sold to the distributor (by the manufacturer) is equal to the total quantity of the product sold by the distributor (to the consumers in the countries A and B). This simplifies to $q_B^{M,D} = q_B^{D,C}$ when parallel trade is forbidden. Finally, in this model $q_A = q_A^{M,C} + q_A^{D,C}$ is the quantity of the product sold to the consumers in the country A, and $q_B = q_B^{D,C}$ is the quantity of the product sold to the consumers in the country B. The process above is illustrated in Figure 1.

3. Optimization of the global welfare function

In this section, using the two-country model for the trade of pharmaceuticals described in Section 2, we first determine the surpluses of the manufacturer, of the distributor, and of the consumers in both countries. Then, on the basis of the obtained formulas, we provide expressions for the global welfare function under three different models for it. Finally, in Subsections 3.1, 3.2, 3.3, respectively, we find in closed form the optimal value of the global welfare function itself (i.e., the

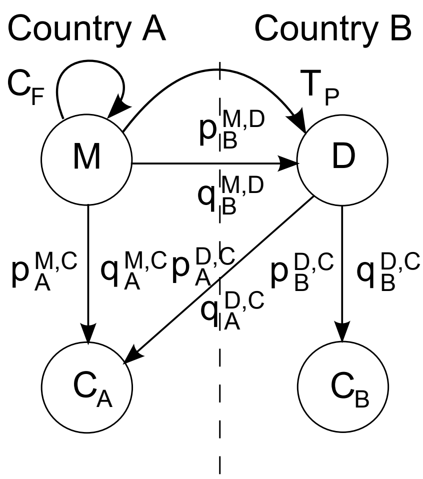


Figure 1: The model of trade among the manufacturer, the distributor, and the consumers in the countries A and B when there is parallel trade freedom. When parallel trade is forbidden, one sets $q_A^{D,C} = 0$.

one that could be found by a hypothetical global planner, by solving a suitable optimization problem), for each of the three models. Then, the results of this analysis are exploited in Section 5 as one of the ingredients needed to compute the price of anarchy for the noncooperative game-theoretic models of parallel trade presented in Section 4.

Proposition 3.1. *For the two-country model of trade of pharmaceuticals described in Section 2, one has the following expressions for the surpluses of the manufacturer, of the distributor, and of the consumers in the countries A and B.*

(a) *When the manufacturer does research and development:*

(i) *Manufacturer's surplus S_M:*

$$S_M = p_A^{M,C} q_A^{M,C} + p_B^{M,D} q_B^{M,D} - C_F + T_P \,. \tag{3}$$

(ii) *Distributor's surplus S_D:*

$$S_D = (p_A^{D,C} - p_B^{M,D} - t) q_A^{D,C} + (p_B^{D,C} - p_B^{M,D}) q_B^{D,C} - T_P \,. \tag{4}$$

(iii) *Consumers' surplus S_{C_A} in the country A:*

$$S_{C_A} = \frac{\gamma a}{b} \left(q_A^{M,C} + q_A^{D,C} \right) - p_A^{M,C} q_A^{M,C} - p_A^{D,C} q_A^{D,C} - \frac{\left(q_A^{M,C} + q_A^{D,C} \right)^2}{2b} \,. \tag{5}$$

(iv) *Consumers' surplus S_{C_B} in the country B:*

$$S_{C_B} = \left(\frac{a}{b} - p_B^{D,C}\right) q_B^{D,C} - \frac{\left(q_B^{D,C}\right)^2}{2b} \,. \tag{6}$$

(b) *When the manufacturer does no research and development:*

(i) *Manufacturer's surplus S_M:*

$$S_M = 0 \,. \tag{7}$$

(ii) *Distributor's surplus S_D:*

$$S_D = 0 \,. \tag{8}$$

(iii) *Consumers' surplus S_{C_A} in the country A:*

$$S_{C_A} = 0 \,. \tag{9}$$

(iv) *Consumers' surplus S_{C_B} in the country B:*

$$S_{C_B} = 0 \,. \tag{10}$$

Proof. *Case (a):* (i), (ii) The proofs of formulas (3) and (4) are immediate.
(iii) In order to prove formula (5), let us suppose that, of the total $q_A = q_A^{M,C} + q_A^{D,C}$ units of the product sold to the consumers in the country A, the first $q_A^{M,C}$ ones are sold by the manufacturer, whereas the last $q_A^{D,C}$ ones are sold by the distributor. Then, taking into account the expression (1) of the demand function in the country A, the consumers' surplus in the country A for the first $q_A^{M,C}$ units of the product is

$$S_{C_A}^{(I)} = \int_0^{q_A^{M,C}} \left(\frac{\gamma a - q}{b} - p_A^{M,C}\right) dq = \left(\frac{\gamma a}{b} - p_A^{M,C}\right) q_A^{M,C} - \frac{\left(q_A^{M,C}\right)^2}{2b} \,. \tag{11}$$

Similarly, the consumers' surplus in the country A for the remaining $q_A^{D,C}$ units of the product is

$$\begin{aligned} S_{C_A}^{(II)} &= \int_{q_A^{M,C}}^{q_A^{M,C}+q_A^{D,C}} \left(\frac{\gamma a - q}{b} - p_A^{D,C}\right) dq \\ &= \left(\frac{\gamma a}{b} - p_A^{D,C}\right) q_A^{D,C} - \frac{\left(q_A^{M,C} + q_A^{D,C}\right)^2}{2b} + \frac{\left(q_A^{M,C}\right)^2}{2b} \,. \end{aligned} \tag{12}$$

Then, formula (5) is obtained by summing (11) and (12). It is important to observe that one obtains exactly the same expression (5) for S_{C_A} if other choices for the seller are considered, for each unit of the product. For instance, if one assumes instead that the first $q_A^{D,C}$ units of the product bought by the consumers in the country A are sold by the distributor, whereas the last $q_A^{M,C}$ ones are sold by the manufacturer, formulas (11) and (12) are replaced, respectively, by

$$S_{C_A}^{(I')} = \int_0^{q_A^{D,C}} \left(\frac{\gamma a - q}{b} - p_A^{D,C} \right) dq = \left(\frac{\gamma a}{b} - p_A^{D,C} \right) q_A^{D,C} - \frac{\left(q_A^{D,C}\right)^2}{2b} , \quad (13)$$

and

$$\begin{aligned} S_{C_A}^{(II')} &= \int_{q_A^{D,C}}^{q_A^{D,C}+q_A^{M,C}} \left(\frac{\gamma a - q}{b} - p_A^{M,C} \right) dq \\ &= \left(\frac{\gamma a}{b} - p_A^{M,C} \right) q_A^{M,C} - \frac{\left(q_A^{D,C} + q_A^{M,C}\right)^2}{2b} + \frac{\left(q_A^{D,C}\right)^2}{2b} , \end{aligned} \quad (14)$$

and also the sum of (13) and (14) provides the expression (5) for S_{C_A}.
(iv) Finally, formula (6) is proved likewise formula (5), noting that the consumers in the country B buy only from the distributor:

$$S_{C_B} = \int_0^{q_B^{D,C}} \left(\frac{a - q}{b} - p_B^{D,C} \right) dq = \left(\frac{a}{b} - p_B^{D,C} \right) q_B^{D,C} - \frac{\left(q_B^{D,C}\right)^2}{2b} . \quad (15)$$

Case (b): (i), (ii), (iii), (iv) When the manufacturer does no research and development, no costs are incurred and no quantities are exchanged, so all the surpluses are equal to 0. ■

We now define the following three models for the global welfare function, to be maximized by the global planner under suitable constraints. In the following, we use the subscript "PT" to denote the situation in which there is parallel trade freedom, whereas the subscript "NPT" refers to the case in which parallel trade is not permitted.

(i) **Bentham model**: it is defined as the sum of all the surpluses. When the manufacturer does research and development and parallel trade is permitted, it has the following expression[2] $GW_{PT}^{(B)}$, which is a function only of the total fixed cost of production and of the traded quantities (for simplicity, here we have removed the dependence on $q_B^{M,D}$, using the constraint $q_B^{M,D} = q_A^{D,C} + q_B^{D,C}$):

$$\begin{aligned}
&GW_{PT}^{(B)}(C_F, q_A^{M,C}, q_A^{D,C}, q_B^{D,C}) \\
=&S_M + S_D + S_{C_A} + S_{C_B} \\
=&\frac{\gamma a}{b}\left(q_A^{M,C} + q_A^{D,C}\right) - \frac{\left(q_A^{M,C} + q_A^{D,C}\right)^2}{2b} - tq_A^{D,C} + \frac{a}{b}q_B^{D,C} - \frac{\left(q_B^{D,C}\right)^2}{2b} - C_F .
\end{aligned} \tag{16}$$

When the manufacturer does research and development and parallel trade is not permitted, it has the following simpler expression $GW_{NPT}^{(B)}$:

$$\begin{aligned}
&GW_{NPT}^{(B)}(C_F, q_A^{M,C}, q_B^{D,C}) \\
=&S_M + S_D + S_{C_A} + S_{C_B} \\
=&\frac{\gamma a}{b}q_A^{M,C} - \frac{\left(q_A^{M,C}\right)^2}{2b} + \frac{a}{b}q_B^{D,C} - \frac{\left(q_B^{D,C}\right)^2}{2b} - C_F ,
\end{aligned} \tag{17}$$

which is obtained from (16) by setting $q_A^{D,C} = 0$.
Finally, when the manufacturer does no research and development, one has obviously $GW_{NPT}^{(B)} = 0$.

(ii) **Rawls model, first specification**: it is defined as the minimum between the national welfares of the two countries, where each national welfare is defined as the sum of the surpluses of the entities belonging to that country. Hence, when the manufacturer does research and development and parallel trade is permitted, it has the following expression $GW_{PT}^{(R,I)}$, which is a function of the total fixed cost of production, of the transfer payment, of the prices involving entities belonging to different countries, and of all the quantities:

[2]Formula (16) is derived using the expressions for S_M, S_D, S_{C_A}, and S_{C_B} provided by formulas (3), (4), (5), and (6), respectively, taking into account the constraint $q_B^{M,D} = q_A^{D,C} + q_B^{D,C}$, and observing that all the terms containing prices cancel out in the summation.

$$
\begin{aligned}
& GW_{PT}^{(R,I)}(C_F, T_P, p_A^{D,C}, p_B^{M,D}, q_A^{M,C}, q_A^{D,C}, q_B^{M,D}, q_B^{D,C}) \\
= \; & \min\{S_M + S_{C_A}, S_D + S_{C_B}\} \\
= \; & \min\Bigg\{ p_B^{M,D} q_B^{M,D} + \frac{\gamma a}{b}\left(q_A^{M,C} + q_A^{D,C}\right) - p_A^{D,C} q_A^{D,C} - \frac{\left(q_A^{M,C} + q_A^{D,C}\right)^2}{2b} - C_F + T_P \, , \\
& \left(p_A^{D,C} - p_B^{M,D} - t\right) q_A^{D,C} + \left(\frac{a}{b} - p_B^{M,D}\right) q_B^{D,C} - \frac{\left(q_B^{D,C}\right)^2}{2b} - T_P \Bigg\} .
\end{aligned}
\tag{18}
$$

When the manufacturer does research and development and parallel trade is not permitted, it has the following simpler expression $GW_{NPT}^{(R,I)}$:

$$
\begin{aligned}
& GW_{NPT}^{(R,I)}(C_F, T_P, p_B^{M,D}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C}) \\
= \; & \min\{S_M + S_{C_A}, S_D + S_{C_B}\} \\
= \; & \min\Bigg\{ p_B^{M,D} q_B^{M,D} + \frac{\gamma a}{b} q_A^{M,C} - \frac{\left(q_A^{M,C}\right)^2}{2b} \, , \\
& \left(\frac{a}{b} - p_B^{M,D}\right) q_B^{D,C} - \frac{\left(q_B^{D,C}\right)^2}{2b} \Bigg\} .
\end{aligned}
\tag{19}
$$

Finally, when the manufacturer does no research and development, one has obviously $GW_{NPT}^{(R,I)} = 0$.

(iii) **Rawls model, second specification**: it is defined as the minimum of all the surpluses. Hence, when the manufacturer does research and development and parallel trade is permitted, it has the following expression $GW_{PT}^{(R,II)}$, which is a function of the total fixed cost of production, of the transfer payment, and of all the prices and quantities:

$$
\begin{aligned}
& GW_{PT}^{(R,II)}(C_F, T_P, p_A^{M,C}, p_A^{D,C}, p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_A^{D,C}, q_B^{M,D}, q_B^{D,C}) \\
= \;& \min\{S_M, S_D, S_{C_A}, S_{C_B}\} \\
= \;& \min\left\{ p_A^{M,C} q_A^{M,C} + p_B^{M,D} q_B^{M,D} - C_F + T_P \, , \right. \\
& (p_A^{D,C} - p_B^{M,D} - t) q_A^{D,C} + (p_B^{D,C} - p_B^{M,D}) q_B^{D,C} - T_P \, , \\
& \frac{\gamma a}{b} \left(q_A^{M,C} + q_A^{D,C} \right) - p_A^{M,C} q_A^{M,C} - p_A^{D,C} q_A^{D,C} - \frac{\left(q_A^{M,C} + q_A^{D,C} \right)^2}{2b} \, , \\
& \left. \left(\frac{a}{b} - p_B^{D,C} \right) q_B^{D,C} - \frac{\left(q_B^{D,C} \right)^2}{2b} \right\} .
\end{aligned}
\tag{20}
$$

When the manufacturer does research and development and parallel trade is not permitted, it has the following simpler expression $GW_{NPT}^{(R,II)}$:

$$
\begin{aligned}
& GW_{NPT}^{(R,II)}(C_F, T_P, p_A^{M,C}, p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C}) \\
= \;& \min\{S_M, S_D, S_{C_A}, S_{C_B}\} \\
= \;& \min\left\{ p_A^{M,C} q_A^{M,C} + p_B^{M,D} q_B^{M,D} - C_F + T_P \, , \right. \\
& (p_B^{D,C} - p_B^{M,D}) q_B^{D,C} - T_P \, , \\
& \left(\frac{\gamma a}{b} - p_A^{M,C} q_A^{M,C} \right) - \frac{\left(q_A^{M,C} \right)^2}{2b} \, , \\
& \left. \frac{a}{b} q_B^{D,C} - p_B^{D,C} q_B^{D,C} - \frac{\left(q_B^{D,C} \right)^2}{2b} \right\} .
\end{aligned}
\tag{21}
$$

Finally, when the manufacturer does no research and development, one has obviously $GW_{NPT}^{(R,II)} = 0$.

In the following subsections, for each model of the global welfare function, we find its optimal value for a global planner who maximizes it under suitable assumptions. In general, two kinds of results are obtained: one when the manufacturer does research and development, the other one when the manufacturer does no research and development. Of the two situations, the global planner prefers the one with the largest value of the global welfare.

Remark 3.2. Since, for each model, the second situation is associated with a 0 value of the global welfare, the optimal global welfare for the global planner is always non-negative. Moreover, an inspection of the proofs in the next subsections shows that the corresponding optimal solution for the global planner is always associated with non-negative surpluses for all the entities involved (manufacturer, distributor, and consumers of both countries). ■

3.1. *Optimization of the global welfare under the Bentham model*

We first consider the case in which the manufacturer does research and development and parallel trade is permitted. Then, in order to find the optimal value of the global welfare $GW_{PT}^{(B)}$ provided by formula (16) under the Bentham model, the global planner has to solve the following optimization problem:

$$\begin{aligned} \underset{q_A^{M,C}, q_A^{D,C}, q_B^{D,C}}{\text{maximize}} \quad & GW_{PT}^{(B)}(C_F, q_A^{M,C}, q_A^{D,C}, q_B^{D,C}) \\ \text{s.t.} \quad & q_A^{M,C}, q_A^{D,C}, q_B^{D,C} \geq 0 \,. \end{aligned} \tag{22}$$

Of course, when the manufacturer does no research and development and parallel trade is permitted, there is nothing to optimize, and the optimal value of $GW_{PT}^{(B)}$ is 0.

Proposition 3.3. *When the manufacturer does research and development: the optimal value of the objective $GW_{PT}^{(B)}$ of the optimization problem (22) modeling parallel trade freedom under the Bentham global welfare model is*

$$\left(GW_{PT}^{(B)}\right)^\circ = \frac{(\gamma^2+1)\,a^2}{2b} - C_F \,. \tag{23}$$

When the manufacturer does no research and development: the optimal value of $GW_{PT}^{(B)}$ is

$$\left(GW_{PT}^{(B)}\right)^\circ = 0 \,. \tag{24}$$

Proof. We start considering the case in which the manufacturer does research and development. One can observe that the global welfare $GW_{PT}^{(B)}$ provided by formula (16) is a function having the separable structure

$$GW_{PT}^{(B)}(C_F, q_A^{M,C}, q_A^{D,C}, q_B^{D,C}) = GW_{PT}^{(B,I)}(q_A^{M,C}, q_A^{D,C}) + GW_{PT}^{(B,II)}(q_B^{D,C}), \quad (25)$$

where

$$GW_{PT}^{(B,I)}(q_A^{M,C}, q_A^{D,C}) = \frac{\gamma a}{b}\left(q_A^{M,C} + q_A^{D,C}\right) - \frac{\left(q_A^{M,C} + q_A^{D,C}\right)^2}{2b} - tq_A^{D,C} - C_F, \quad (26)$$

and

$$GW_{PT}^{(B,II)}(q_B^{D,C}) = \frac{a}{b}q_B^{D,C} - \frac{\left(q_B^{D,C}\right)^2}{2b}. \quad (27)$$

Hence, due to the separability of its objective function and to the form of its constraints, solving the optimization problem (22) is reduced to solving the two following optimization problems:

$$\begin{aligned} \underset{q_A^{M,C}, q_A^{D,C}}{\text{maximize}} \quad & GW_{PT}^{(B,I)}(q_A^{M,C}, q_A^{D,C}) = \frac{\gamma a}{b}\left(q_A^{M,C} + q_A^{D,C}\right) - \frac{\left(q_A^{M,C} + q_A^{D,C}\right)^2}{2b} - tq_A^{D,C} - C_F \\ \text{s. t.} \quad & q_A^{M,C}, q_A^{D,C} \geq 0, \end{aligned} \quad (28)$$

and

$$\begin{aligned} \underset{q_B^{D,C}}{\text{maximize}} \quad & GW_{PT}^{(B,II)}(q_B^{D,C}) = \frac{a}{b}q_B^{D,C} - \frac{\left(q_B^{D,C}\right)^2}{2b} \\ \text{s. t.} \quad & q_B^{D,C} \geq 0. \end{aligned} \quad (29)$$

We now consider the two problems separately.

(i) The optimization problem (28) is a concave quadratic maximization problem. By introducing the Lagrangian function

$$L^{(I)}(q_A^{M,C}, q_A^{D,C}, \mu_A^{M,C}, \mu_A^{D,C}) = GW_{PT}^{(B,I)}(q_A^{M,C}, q_A^{D,C}) + \mu_A^{M,C} q_A^{M,C} + \mu_A^{D,C} q_A^{D,C}, \quad (30)$$

it is solved by imposing the following Karush-Kuhn-Tucker optimality conditions[3]:

$$
\begin{cases}
stationarity: & \dfrac{\partial L^{(I)}}{\partial q_A^{M,C}} = \dfrac{\gamma a}{b} - \dfrac{q_A^{M,C} + q_A^{D,C}}{b} + \mu_A^{M,C} = 0\,, \\
& \dfrac{\partial L^{(I)}}{\partial q_A^{D,C}} = \dfrac{\gamma a}{b} - \dfrac{q_A^{M,C} + q_A^{D,C}}{b} - t + \mu_A^{D,C} = 0\,, \\
primal\,feasibility: & q_A^{M,C}, q_A^{D,C} \geq 0\,, \\
dual\,feasibility: & \mu_A^{M,C}, \mu_A^{D,C} \geq 0\,, \\
complementary\,slackness: & \mu_A^{M,C} q_A^{M,C}, \mu_A^{D,C} q_A^{D,C} = 0\,.
\end{cases}
\tag{31}
$$

Then, it is straightforward to see that, for any $t > 0$, the system (31) has the unique solution

$$
\begin{cases}
q_A^{M,C} = \gamma a\,, \\
q_A^{D,C} = 0\,, \\
\mu_A^{M,C} = 0\,, \\
\mu_A^{D,C} = t\,.
\end{cases}
\tag{32}
$$

For $t = 0$, one gets the infinite number of solutions described by

$$
\begin{cases}
q_A^{M,C}, q_A^{D,C} \geq 0 \text{ s. t. } q_A^{M,C} + q_A^{D,C} = \gamma a\,, \\
\mu_A^{M,C} = 0\,, \\
\mu_A^{D,C} = 0\,.
\end{cases}
\tag{33}
$$

Finally, for both formulas (32) and (33), one has $q_A = q_A^{M,C} + q_A^{D,C} = \gamma a$. Hence, for both cases, the value of the objective $GW_{PT}^{(B,I)}(q_A^{M,C}, q_A^{D,C})$ at optimality is

$$
\left(GW_{PT}^{(B,I)}\right)^{\circ} = \frac{(\gamma a)^2}{b} - C_F - \frac{(\gamma a)^2}{2b} = \frac{(\gamma a)^2}{2b} - C_F\,. \tag{34}
$$

[3] For both optimization problems (28) and (29), the qualification of the constraints holds, due to their linearity [10]. Moreover, for both problems, Karush-Kuhn-Tucker optimality conditions are necessary and sufficient for optimality, due to the concavity of the respective objective functions.

(ii) Similarly, also the optimization problem (29) is a concave quadratic maximization problem. Again, by introducing the Lagrangian function

$$L^{(II)}(q_B^{D,C}, \mu_B^{D,C}) = \frac{a}{b} q_B^{D,C} - \frac{\left(q_B^{D,C}\right)^2}{2b}\,, \tag{35}$$

also the optimization problem (29) is solved by imposing the Karush-Kuhn-Tucker optimality conditions, which have now the following form:

$$\begin{cases} stationarity: & \dfrac{\partial L^{(II)}}{\partial q_B^{D,C}} = \dfrac{a}{b} - \dfrac{q_B^{D,C}}{b} + \mu_B^{D,C} = 0\,, \\ primal\,feasibility: & q_B^{D,C} \geq 0\,, \\ dual\,feasibility: & \mu_B^{D,C} \geq 0\,, \\ complementary\,slackness: & \mu_B^{D,C} q_B^{D,C} = 0\,. \end{cases} \tag{36}$$

Then, one can see that, for any $t \geq 0$, the system (36) has the unique solution

$$\begin{cases} q_B^{D,C} = a\,, \\ \mu_B^{D,C} = 0\,. \end{cases} \tag{37}$$

Finally, the value of the objective $GW_{PT}^{(B,I)}(q_B^{D,C})$ at optimality is

$$\left(GW_{PT}^{(B,II)}\right)^{\circ} = \frac{a^2}{b} - \frac{a^2}{2b} = \frac{a^2}{2b}\,. \tag{38}$$

Concluding, the optimal value of the objective $GW_{PT}^{(B)}(q_A^{M,D}, q_A^{D,C}, q_B^{D,C})$ of the original optimization problem (22) modeling parallel trade freedom under the Bentham global welfare model is

$$\left(GW_{PT}^{(B)}\right)^{\circ} = \left(GW_{PT}^{(B,I)}\right)^{\circ} + \left(GW_{PT}^{(B,II)}\right)^{\circ} = \frac{(\gamma^2+1)\,a^2}{2b} - C_F\,,$$

which is (23). Finally, for the case in which the manufacturer does no research and development, (24) follows trivially from the definition of the present global welfare function in Section 3. ∎

Remark 3.4. The meaning of the optimal solutions of the optimization problems (28) and (29) considered in the proof of Proposition 3.3 is the following. Since there is no dependence of their objectives on the prices, one can assume at first that all the prices are equal to 0. In such a case, the surpluses of the manufacturer and of the distributor would be $-C_F$ and 0, respectively. Moreover, at optimality, the consumers in each country would obtain the maximum desired quantity of the product (i.e., taking into account the expressions (1) and (2) of the respective demand functions, γa in the country A, and a in the country B). As a consequence, using formula (5), the corresponding surplus for the consumers in the country A would be

$$S_{C_A} = \frac{\gamma a}{b}(\gamma a) - \frac{(\gamma a)^2}{2b} = \frac{(\gamma a)^2}{2b}, \tag{39}$$

whereas, using formula (6), the corresponding surplus for the consumers in the country B would be

$$S_{C_B} = \frac{a}{b}a - \frac{a^2}{2b} = \frac{a^2}{2b}. \tag{40}$$

Hence, their sum would be equal to (23). For the case of non-zero prices, the optimal sum of all the surpluses would be the same as in (23), but it would be re-distributed among the manufacturer, the distributor, and the consumers in the two countries. ■

For the case in which parallel trade is forbidden, one has to use the expression (17) for the Bentham global welfare function $GW_{NPT}^{(B)}$, and solve the following optimization problem:

$$\begin{aligned} \underset{q_A^{M,C}, q_B^{D,C}}{\text{maximize}} \quad & GW_{NPT}^{(B)}(C_F, q_A^{M,C}, q_B^{D,C}) \\ \text{s.t.} \quad & q_A^{M,C}, q_B^{D,C} \geq 0. \end{aligned} \tag{41}$$

Again, when the manufacturer does no research and development and parallel trade is forbidden, there is nothing to optimize, and the optimal value of $GW_{NPT}^{(B)}$ is 0.

Proposition 3.5. *When the manufacturer does research and development: the optimal value of the objective $GW_{NPT}^{(B)}$ of the optimization problem (41) modeling parallel trade banning under the Bentham global welfare model is*

$$\left(GW_{NPT}^{(B)}\right)^{\circ} = \frac{(\gamma^2+1)\,a^2}{2b} - C_F. \tag{42}$$

When the manufacturer does no research and development: the optimal value of $GW_{NPT}^{(B)}$ is

$$\left(GW_{NPT}^{(B)}\right)^{\circ} = 0\,. \tag{43}$$

Proof. As shown in the proof of Proposition 3.5, for any $t \geq 0$, among the optimal solutions of the optimization problem (22), there is always one for which $q_A^{D,C} = 0$, which is feasible for the more constrained optimization problem (41) (i.e., it satisfies all its constraints). Hence, one obtains (42), whereas (43) follows trivially from the definition of the present global welfare function in Section 3. ■

3.2. Optimization of the global welfare under the first specification of the Rawls model

We first consider the case in which the manufacturer does research and development, there is no transfer payment, and there is parallel trade freedom. Then, by using the expression (18) for the global welfare $GW_{PT}^{(R,I)}$ under the first specification of the Rawls model, and introducing the reduced row vector of prices $\underline{p}_{\text{red}} = \left(p_A^{D,C}, p_B^{M,D}\right)$ and the row vector of quantities $\underline{q} = \left(q_A^{M,C}, q_A^{D,C}, q_B^{M,D}, q_B^{D,C}\right)$, the optimization problem to be solved by the global planner for this case is formulated as

$$\begin{aligned}
\underset{T_P, \underline{p}_{\text{red}}, p_B^{D,C}, \underline{q}}{\text{maximize}} \quad & GW_{PT}^{(R,I)}(C_F, T_P, \underline{p}_{\text{red}}, \underline{q}) \\
\text{s. t.} \quad & q_B^{M,D} = q_A^{D,C} + q_B^{D,C}\,, \\
& p_A^{D,C} \geq p_B^{M,D} + t\,, \\
& p_B^{D,C} \geq p_B^{M,D}\,, \\
& \underline{p}_{\text{red}} \geq \underline{0}, p_B^{D,C} \geq 0, \underline{q} \geq \underline{0}\,, \\
& T_P = 0\,,
\end{aligned} \tag{44}$$

which is a maximin optimization problem. A related maximin optimization problem is

$$\begin{aligned} \underset{T_P, \underline{p}_{\text{red}}, p_B^{D,C}, \underline{q}}{\text{maximize}} \quad & GW_{PT}^{(R,I)}(C_F, T_P, \underline{p}_{\text{red}}, \underline{q}) \\ \text{s. t.} \quad & q_B^{M,D} = q_A^{D,C} + q_B^{D,C}\,, \\ & p_A^{D,C} \geq p_B^{M,D} + t\,, \\ & p_B^{D,C} \geq p_B^{M,D}\,, \\ & \underline{p}_{\text{red}} \geq \underline{0}, p_B^{D,C} \geq 0, \underline{q} \geq \underline{0}\,, \\ & 0 \leq T_P \leq S_D\,, \end{aligned} \tag{45}$$

which is obtained by replacing the constraint $T_P = 0$ in (44) by $0 \leq T_P \leq S_D$ (i.e., a possibly non-zero transfer payment is paid by the distributor to the manufacturer).

Proposition 3.6. *When the manufacturer does research and development: the optimal value of the objective $GW_{PT}^{(R,I)}$ of the optimization problems (44) and (45) modeling parallel trade freedom under the first specification of the Rawls global welfare model is*

$$\left(GW_{PT}^{(R,I)}\right)^{\circ} = \begin{cases} \dfrac{(\gamma^2+1)\,a^2}{4b} - \dfrac{C_F}{2} = \dfrac{1}{2}\left(GW_{PT}^{(B)}\right)^{\circ} & \text{if } \dfrac{(\gamma a)^2}{2b} - C_F \geq -\dfrac{a^2}{2b}\,, \\ \dfrac{(\gamma^2+1)\,a^2}{2b} - C_F < 0 & \text{if } \dfrac{(\gamma a)^2}{2b} - C_F < -\dfrac{a^2}{2b}\,, \end{cases} \tag{46}$$

When the manufacturer does no research and development: the optimal value of $GW_{PT}^{(R,I)}$ is

$$\left(GW_{PT}^{(R,I)}\right)^{\circ} = 0\,. \tag{47}$$

Proof. We first consider the case of the optimization problem (44), when the manufacturer does research and development. Since the global welfare $GW_{PT}^{(R,I)}$ is the minimum between the national welfare of the country A and the national welfare of the country B, and the maximum value of their sum is $\left(GW_{PT}^{(B)}\right)^{\circ}$, one gets

$$\left(GW_{PT}^{(R,I)}\right)^{\circ} \leq \frac{1}{2}\left(GW_{PT}^{(B)}\right)^{\circ}\,. \tag{48}$$

To prove (46), it remains to show that, for every sufficiently small $\varepsilon > 0$, there exists a feasible solution $\left\{C_F, 0, \underline{p}_{\text{red}}^{(\varepsilon)}, (p_B^{D,C})^{(\varepsilon)}, \underline{q}^{(\varepsilon)}\right\}$ of the optimization problem (44) for which one has

$$GW_{PT}^{(R,I)}(C_F, 0, \underline{p}_{\text{red}}^{(\varepsilon)}, \underline{q}^{(\varepsilon)}) \geq \frac{1}{2}\left(GW_{PT}^{(B)}\right)^{\circ} - \varepsilon\,. \tag{49}$$

To obtain such a feasible solution $\left\{C_F, 0, \underline{p}_{\text{red}}^{(\varepsilon)}, (p_B^{D,C})^{(\varepsilon)}, \underline{q}^{(\varepsilon)}\right\}$, we start by considering the case in which $p_A^{D,C} = t$, $p_B^{M,D} = p_B^{D,C} = 0$, $q_A^{M,C} = \gamma a$, $q_A^{D,C} = 0$, $q_B^{M,D} = a$, and $q_B^{D,C} = a$, which corresponds to the feasible solution

$$\left\{C_F, T_P, \underline{\hat{p}}_{\text{red}}, \hat{p}_B^{D,C}, \underline{\hat{q}}\right\} = \{C_F, 0, (t, 0), 0, (\gamma a, 0, a, a)\}\,. \tag{50}$$

In such a situation, one has $GW_{PT}^{(B)}(C_F, 0, \underline{\hat{p}}_{\text{red}}, \underline{\hat{q}}) = \left(GW_{PT}^{(B)}\right)^{\circ}$ (as shown in the proof of Proposition 3.3), and, when $\frac{(\gamma a)^2}{2b} - C_F > \frac{a^2}{2b}$,

$$GW_{PT}^{(R,I)}(C_F, 0, \underline{\hat{p}}_{\text{red}}, \underline{\hat{q}}) = \min\left\{\frac{(\gamma a)^2}{2b} - C_F, \frac{a^2}{2b}\right\} = \frac{a^2}{2b}\,. \tag{51}$$

Then, starting from the feasible solution (50) - for which the national welfare in the country A is greater than the national welfare in the country B - one can transfer part of the current national welfare from the country A to the country B, by increasing the price $p_A^{D,C}$ of an amount $\Delta p > 0$ and the quantity $q_A^{D,C}$ of an amount $\Delta q \in (0, \gamma a]$, and decreasing the quantity $q_A^{M,C}$ of the same amount Δq. In this way, the sum of the two national welfares decreases of $t\Delta q$ (as the quantities $q_A^{M,C} + q_A^{D,C}$ and $q_B^{D,C}$ are kept constant, but a decrease of $t\Delta q$ is incurred, due to the parallel trade costs). More specifically, the national welfare of the country A decreases of $(\Delta p + t)\Delta q$, whereas the national welfare of the country B increases of $\Delta p \Delta q$. Now, we choose Δp and Δq in such a way that $(\Delta p + t)\Delta q = \frac{(\gamma^2-1)a^2}{4b} - \frac{C_F}{2}$ and $t\Delta q = \varepsilon$ (i.e., $\Delta q = \frac{\varepsilon}{t}$ and $\Delta p = \left(\frac{(\gamma^2-1)a^2}{4b} - \frac{C_F}{2} - \varepsilon\right)t$), and we define the new feasible solution

$$\begin{aligned} &\left\{C_F, 0, \underline{p}_{\text{red}}^{(\varepsilon)}, (p_B^{D,C})^{(\varepsilon)}, \underline{q}^{(\varepsilon)}\right\} \\ = &\left\{C_F, 0, \left(\left(1 + \frac{(\gamma^2-1)a^2}{4b} - \frac{C_F}{2} - \varepsilon\right)t, 0\right), 0, \left(\gamma a - \frac{\varepsilon}{t}, \frac{\varepsilon}{t}, a, a\right)\right\}\,. \end{aligned} \tag{52}$$

By construction, $\underline{p}_{\text{red}}^{(\varepsilon)}$ and $\underline{q}^{(\varepsilon)}$ satisfy (49), which completes the proof of (46), taking the limit as ε tends to 0, for the case in which $\frac{(\gamma a)^2}{2b} - C_F > \frac{a^2}{2b}$.

When $\left|\frac{(\gamma a)^2}{2b} - C_F\right| \leq \frac{a^2}{2b}$, formula (51) changes to

$$GW_{PT}^{(R,I)}(\hat{\underline{p}}_{\text{red}}, \hat{\underline{q}}) = \min\left\{\frac{(\gamma a)^2}{2b} - C_F, \frac{a^2}{2b}\right\} = \frac{(\gamma a)^2}{2b} - C_F\,. \tag{53}$$

Also in this case, one can transfer part of the current national welfare from the country B to the country A, by raising the price $p_B^{M,D}$ and $p_B^{D,C}$ of the same amount $\Delta p = \frac{(1-\gamma^2)a^2}{2ab} + \frac{C_F}{a}$, allowing one to find again two choices of $\underline{p}_{\text{red}}^{(\varepsilon)}$ and $\underline{q}^{(\varepsilon)}$ that satisfy (49), hence proving (46) also for this case.

As a last case, when $\frac{(\gamma a)^2}{2b} - C_F < -\frac{a^2}{2b}$, the national welfare in the country A is always negative, whereas the national welfare in the country B is always non-negative, so, starting from the feasible solution (50), one can transfer all the current positive national welfare from the country B to the country A, by raising the price $p_B^{M,D}$ and $p_B^{D,C}$ of the same amount $\Delta p = \frac{a^2}{2ab}$. In this case, the first country obtains its maximum possible (negative) value of the national welfare, which is $\frac{(\gamma^2+1)a^2}{2b} - C_F$.

For the optimization problem (45), one obtains the same expressions as above of the optimal value of the objective $GW_{PT}^{(R,I)}$, due to the fact that the optimal value of the objective in (44) satisfies with equality the upper bound (48) - which is valid also for the problem (45) - and the fact that (45) is less constrained than (44), hence the optimal value of its objective is larger than or equal to the optimal value of the same objective in (44).

Finally, for the case in which the manufacturer does no research and development, (47) follows trivially from the definition of the present global welfare function in Section 3. ■

For the case in which the manufacturer does research and development, there is no transfer payment, and parallel trade is forbidden, one has to use the expression (19) for the global welfare function $GW_{NPT}^{(R,I)}$ under the first specification of the

Rawls model, and solve the following optimization problem:

$$\begin{array}{ll} \underset{T_P, p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C}}{\text{maximize}} & GW_{NPT}^{(R,I)}(C_F, T_P, p_B^{M,D}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C}) \\ \text{s. t.} & q_B^{M,D} = q_B^{D,C}\,, \\ & p_B^{D,C} \geq p_B^{M,D}\,, \\ & p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C} \geq 0\,, \\ & T_P = 0\,, \end{array} \tag{54}$$

which is also a maximin optimization problem. Again, a related maximin optimization problem is

$$\begin{array}{ll} \underset{T_P, p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C}}{\text{maximize}} & GW_{NPT}^{(R,I)}(C_F, T_P, p_B^{M,D}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C}) \\ \text{s. t.} & q_B^{M,D} = q_B^{D,C}\,, \\ & p_B^{D,C} \geq p_B^{M,D}\,, \\ & p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C} \geq 0\,, \\ & 0 \leq T_P \leq S_D\,, \end{array} \tag{55}$$

which is obtained by replacing the constraint $T_P = 0$ in (54) by $0 \leq T_P \leq S_D$.

Proposition 3.7. *When the manufacturer does research and development: the optimal value of the objective $GW_{NPT}^{(R,I)}$ of the optimization problem (54) modeling parallel trade freedom under the first specification of the Rawls global welfare model is*

$$\left(GW_{NPT}^{(R,I)}\right)^{\circ} = \begin{cases} \min\left\{\dfrac{(\gamma a)^2}{2b} - C_F, \dfrac{a^2}{2b}\right\} = \dfrac{a^2}{2b} & \text{if } \dfrac{(\gamma a)^2}{2b} - C_F > \dfrac{a^2}{2b}\,, \\ \dfrac{(\gamma^2+1)\,a^2}{4b} - \dfrac{C_F}{2} = \dfrac{1}{2}\left(GW_{PT}^{(B)}\right)^{\circ} & \text{if } \left|\dfrac{(\gamma a)^2}{2b} - C_F\right| \leq \dfrac{a^2}{2b}\,, \\ \dfrac{(\gamma^2+1)\,a^2}{2b} - C_F < 0 & \text{if } \dfrac{(\gamma a)^2}{2b} - C_F < -\dfrac{a^2}{2b}\,, \end{cases} \tag{56}$$

whereas the corresponding optimal value of the objective $GW_{NPT}^{(R,I)}$ of the optimization problem (55) is

$$\left(GW_{NPT}^{(R,I)}\right)^{\circ} = \frac{(\gamma^2+1)\,a^2}{4b} - \frac{C_F}{2} = \frac{1}{2}\left(GW_{PT}^{(B)}\right)^{\circ}\,. \tag{57}$$

When the manufacturer does no research and development: the optimal value of $GW_{NPT}^{(R,I)}$ is

$$\left(GW_{NPT}^{(R,I)}\right)^{\circ} = 0\,. \tag{58}$$

Proof. We first consider the case of the optimization problem (54), when the manufacturer does research and development. When $\frac{(\gamma a)^2}{2b} - C_F > \frac{a^2}{2b}$, starting from the feasible solution

$$(C_F, T_P, \bar{p}_B^{M,D}, \bar{q}_A^{M,C}, \bar{q}_B^{M,D}, \bar{q}_B^{D,C}) = (C_F, 0, 0, \gamma a, a, a)\ , \tag{59}$$

which corresponds to the one $\{C_F, 0, (t, 0), 0, (\gamma a, 0, a, a)\}$ in (50) and produces the same values for the national welfares (which also maximize the sum of the national welfares, as already shown in the proof of Proposition 3.3), it is not possible to transfer part of the current national welfare from the country A to the country B. Indeed, a negative price $p_B^{M,D}$ is not admissible, and, differently from the proof of Proposition 3.7, one cannot increase $q_A^{D,C}$, which has to be equal to 0 due to the assumption of parallel trade banning. Then, since its set of feasible solutions is convex, (59) is an optimal solution of (54), which proves (56) for $\frac{(\gamma a)^2}{2b} - C_F \geq \frac{a^2}{2b}$.

When $\left|\frac{(\gamma a)^2}{2b} - C_F\right| \leq \frac{a^2}{2b}$, starting from the feasible solution (59), one can transfer part of the current national welfare from the country B to the country A, by raising the price $p_B^{M,D}$ and $p_B^{D,C}$ of the same amount $\Delta p = \frac{(1-\gamma^2)a^2}{2ab} + \frac{C_F}{a}$, allowing one to conclude as in the corresponding part of the proof of Proposition 3.6.

As a last case, when $\frac{(\gamma a)^2}{2b} - C_F < -\frac{a^2}{2b}$, the national welfare in the country A is always negative, whereas the national welfare in the country B is always non-negative, so, starting from the feasible solution (59), one can transfer all the current positive national welfare from the country B to the country A, by raising the price $p_B^{M,D}$ and $p_B^{D,C}$ of the same amount $\Delta p = \frac{a^2}{2ab}$. In this case, the first country obtains its maximum possible (negative) value of the national welfare, which is $\frac{(\gamma^2+1)a^2}{2b} - C_F$.

For the optimization problem (54), the only difference with respect to the proof above is that, when the manufacturer does research and development and $\frac{(\gamma a)^2}{2b} - C_F \geq \frac{a^2}{2b}$, starting from the feasible solution (59), one can transfer part of the current national welfare from the country B to the country A, by raising the transfer payment from 0 to $\frac{(1-\gamma^2)a^2}{4b} + \frac{C_F}{2}$, making the two national welfares equal to the same value $\frac{(\gamma^2+1)a^2}{4b} - \frac{C_F}{2} = \frac{1}{2}\left(GW_{PT}^{(B)}\right)^{\circ}$.

Finally, for the case in which the manufacturer does no research and development, (58) follows trivially from the definition of the present global welfare function in Section 3. ■

3.3. *Optimization of the global welfare under the second specification of the Rawls model*

Again, we first consider the case in which the manufacturer does research and development, there is no transfer payment, and there is parallel trade freedom. Then, by using the expression (20) for the global welfare $GW_{PT}^{(R,II)}$ under the second specification of the Rawls model, and introducing the row vector of prices $\underline{p} = \left(p_A^{M,C}, p_A^{D,C}, p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_A^{D,C}, q_B^{M,D}, q_B^{D,C}\right)$ besides the row vector of quantities $\underline{q} = \left(q_A^{M,C}, q_A^{D,C}, q_B^{M,D}, q_B^{D,C}\right)$ already used in Subsection 3.2, the optimization problem to be solved by the global planner is now formulated as

$$\begin{aligned} \underset{T_P, \underline{p}, \underline{q}}{\text{maximize}} \quad & GW_{PT}^{(R,II)}(C_F, T_P, \underline{p}, \underline{q}) \\ \text{s. t.} \quad & q_B^{M,D} = q_A^{D,C} + q_B^{D,C}\,, \\ & p_A^{D,C} \geq p_B^{M,D} + t\,, \\ & p_B^{D,C} \geq p_B^{M,D}\,, \\ & \underline{p}, \underline{q} \geq \underline{0}\,, \\ & T_P = 0\,, \end{aligned} \tag{60}$$

which is a maximin optimization problem, likewise the optimization problem (44). Again, a related maximin optimization problem is

$$\begin{aligned} \underset{T_P, \underline{p}, \underline{q}}{\text{maximize}} \quad & GW_{PT}^{(R,II)}(C_F, T_P, \underline{p}, \underline{q}) \\ \text{s. t.} \quad & q_B^{M,D} = q_A^{D,C} + q_B^{D,C}\,, \\ & p_A^{D,C} \geq p_B^{M,D} + t\,, \\ & p_B^{D,C} \geq p_B^{M,D}\,, \\ & \underline{p}, \underline{q} \geq \underline{0}\,, \\ & 0 \leq T_P \leq S_D\,, \end{aligned} \tag{61}$$

which is obtained by replacing the constraint $T_P = 0$ in (60) by $0 \leq T_P \leq S_D$.

Proposition 3.8. *When the manufacturer does research and development: the optimal value of the objective* $GW_{PT}^{(R,II)}$ *of the optimization problems (60) and (61) modeling parallel trade freedom under the second specification of the Rawls global welfare model is*

$$\left(GW_{PT}^{(R,II)}\right)^{\circ} = \begin{cases} \frac{a^2}{2b} & \text{if } \gamma > \sqrt{3+\frac{2bC_F}{a^2}}\,, \\ \frac{(\gamma^2+1)a^2}{8b} - \frac{C_F}{4} = \frac{1}{4}\left(GW_{PT}^{(B)}\right)^{\circ} & \text{if } 1 < \gamma \leq \sqrt{3+\frac{2bC_F}{a^2}} \\ & \text{and } \frac{(\gamma a)^2}{2b} - C_F \geq -\frac{a^2}{2b}\,, \\ \frac{(\gamma^2+1)a^2}{2b} - \frac{C_F}{2} & \text{if } \frac{(\gamma a)^2}{2b} - C_F < -\frac{a^2}{2b}\,. \end{cases}$$

When the manufacturer does no research and development: the optimal value of $GW_{NPT}^{(R,II)}$ *is*

$$\left(GW_{NPT}^{(R,II)}\right)^{\circ} = 0\,. \tag{62}$$

Proof. We start considering the case in which the manufacturer does research and development (the following analysis is the same for both problems (60) and (61)). By Proposition 3.3, the optimal value of the sum of all the surpluses is equal to $\left(GW_{PT}^{(B)}\right)^{\circ} = \frac{(\gamma^2+1)a^2}{2b} - C_F$, which is also achieved according to its proof. When $1 < \gamma \leq \sqrt{3+\frac{2bC_F}{a^2}}$ and $\frac{(\gamma a)^2}{2b} - C_F \geq -\frac{a^2}{2b}$, by choosing a suitable feasible vector of prices $\underline{p}$, one can re-distribute the surpluses corresponding to such an optimal solution equally among the 4 entities, thus obtaining

$$\left(GW_{PT}^{(R,II)}\right)^{\circ} = \frac{1}{4}\left(GW_{PT}^{(B)}\right)^{\circ} = \frac{(\gamma^2+1)\,a^2}{8b} - \frac{C_F}{4}\,. \tag{63}$$

When $\gamma > \sqrt{3+\frac{2bC_F}{a^2}}$, instead, by choosing another suitable feasible vector of prices $\underline{p}$ with $p_B^{M,D} = p_B^{D,C} = 0$, one can achieve the value $\frac{a^2}{2b}$ for the surplus S_{C_B} of the consumers in the country B (which is also its maximum value, as it can be seen by observing that S_{C_B} is bounded from above by the optimal value of the objective of the optimization problem (29)) and re-distribute the remaining sum

$\frac{\gamma^2 a^2}{2b} - C_F > 3\frac{a^2}{2b}$ of the surpluses equally among the manufacturer, the distributor, and the consumers in the country A. Hence, for such a case, one obtains

$$\left(GW_{PT}^{(R,II)}\right)^{\circ} = \frac{a^2}{2b}\,. \tag{64}$$

As a last case, when $\frac{(\gamma a)^2}{2b} - C_F < -\frac{a^2}{2b}$, the surplus of the manufacturer is always negative, whereas the surpluses of all the other entities are always nonnegative, so, starting from the feasible solution (59), one can transfer all the current positive surpluses of the other entities to the manufacturer, by raising the price $p_B^{M,D}$ and $p_B^{D,C}$ of the same amount $\Delta p = \frac{a^2}{2ab}$. In this case, the manufacturer obtains its maximum possible (negative) value of the surplus, which is $\frac{\left(\gamma^2+1\right)a^2}{2b} - C_F$.

Finally, for the case in which the manufacturer does no research and development, (62) follows trivially from the definition of the present global welfare function in Section 3. ■

As a last case, when the manufacturer does research and development, there is no transfer payment, and parallel trade is forbidden, one has to use the expression (21) for the global welfare function $GW_{NPT}^{(R,II)}$ under the second specification of the Rawls model, and solve the following optimization problem:

$$\begin{aligned}
\underset{T_P, p_A^{M,C}, p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C}}{\text{maximize}} \quad & GW_{NPT}^{C_F,T_P,(R,II)}(p_A^{M,C}, p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C}) \\
\text{s. t.} \quad & q_B^{M,D} = q_B^{D,C}\,, \\
& p_B^{D,C} \geq p_B^{M,D}\,, \\
& p_A^{M,C}, p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C} \geq 0\,, \\
& T_P = 0\,,
\end{aligned} \tag{65}$$

which is also a maximin optimization problem. Again, a related maximin optimization problem is

$$\begin{aligned}
\underset{T_P, p_A^{M,C}, p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C}}{\text{maximize}} \quad & GW_{NPT}^{C_F,T_P,(R,II)}(p_A^{M,C}, p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C}) \\
\text{s. t.} \quad & q_B^{M,D} = q_B^{D,C}\,, \\
& p_B^{D,C} \geq p_B^{M,D}\,, \\
& p_A^{M,C}, p_B^{M,D}, p_B^{D,C}, q_A^{M,C}, q_B^{M,D}, q_B^{D,C} \geq 0\,, \\
& 0 \leq T_P \leq S_D\,,
\end{aligned} \tag{66}$$

which is obtained by replacing the constraint $T_P = 0$ in (65) by $0 \leq T_P \leq S_D$.

Proposition 3.9. *When the manufacturer does research and development: the optimal value of the objective $GW_{NPT}^{(R,II)}$ of the optimization problem (65) modeling parallel trade freedom under the second specification of the Rawls global welfare model is*

$$\left(GW_{NPT}^{(R,II)}\right)^{\circ} = \begin{cases} \frac{a^2}{4b} = \frac{1}{2}\left(GW_{NPT}^{(R,I)}\right)^{\circ} & \text{if } \frac{(\gamma a)^2}{2b} - C_F > \frac{a^2}{2b}, \\ \frac{\left(\gamma^2+1\right)a^2}{8b} - \frac{C_F}{4} = \frac{1}{4}\left(GW_{PT}^{(B)}\right)^{\circ} & \text{if } \left|\frac{(\gamma a)^2}{2b} - C_F\right| \leq \frac{a^2}{2b}, \\ \frac{\left(\gamma^2+1\right)a^2}{2b} - C_F < 0 & \text{if } \frac{(\gamma a)^2}{2b} - C_F < -\frac{a^2}{2b}. \end{cases} \tag{67}$$

Proof. We first consider the case of the optimization problem (65), when the manufacturer does research and development. When $\frac{(\gamma a)^2}{2b} - C_F \geq -\frac{a^2}{2b}$, starting from an optimal solution of the optimization problem (54) (which maximizes the minimum of the two national welfares), for each country, the global planner can re-distribute the national welfare equally between the two entities of the country, by making suitable feasible choices for the prices. Hence, taking into account the first two cases of formula (56), one obtains the first two cases of formula (67). Finally, when $\frac{(\gamma a)^2}{2b} - C_F < -\frac{a^2}{2b}$, one can proceed likewise in the corresponding part of the proof of Proposition 3.7, hence proving also the last case of formula (67). ∎

4. Background: three game-theoretic models for parallel trade banning/parallel trade threat/parallel trade occurrence

In practice, the prices and quantities of the model presented in Section 2 in general cannot be chosen realistically by a global planner, because - given the demand functions - they depend on the interaction between the manufacturer and the distributor. As already-mentioned in Section 1, two game-theoretic dynamic noncooperative models were proposed in [7] to describe the interaction between the manufacturer and the distributor, and another one was proposed earlier in [5]. Although the three models above refer to increasing levels of complexity, their subgame-perfect Nash equilibria[4] determine prices and quantities at equilibrium that are in general different from the ones determined by the global planner. In this section, we shortly summarize such game-theoretic models. Then, in Section 5 , we compute the corresponding value of the global welfare function at equilibrium, for each of the three models of the global welfare function presented in Section 3. Finally, in Section 5, by using the concept of price of anarchy, we compute the loss in efficiency in the optimization of the global welfare function, which is incurred when moving from the optimal solution determined by the global planner to the prices and quantities at the "worst" subgame-perfect Nash equilibrium.

The following is a short summary of the results of the analysis of the noncooperative game-theoretic models proposed in [5] and [7] for the interaction among the manufacturer and the distributor. The models are ordered according to an increasing level of complexity of the interaction between the manufacturer and the distributor.

(i) **First noncooperative game-theoretic model: parallel trade is forbidden**, i.e., $q_A^{D,C} = 0$. The interaction of the players (here, the manufacturer and the distributor) is described by a dynamic noncooperative game with perfect and complete information. In the first stage, the manufacturer sets the wholesale price $p_B^{M,D}$ for the distributor. Then, in the second stage, the distributor sets the retail price $p_B^{D,C}$ for the consumers in the country B. No transfer payment is paid by the distributor to the manufacturer. The game is solved in [12] by backward induction (the final result is also reported in [7]), providing the following prices and quantities at a subgame-perfect Nash equilibrium:

[4] We recall that a subgame-perfect Nash equilibrium of a dynamic noncooperative game is an equilibrium such that its players' strategies constitute a Nash equilibrium for every subgame of the original dynamic noncooperative game. Not all Nash equilibria are also subgame-perfect Nash equilibria. The difference between a generic Nash equilibrium and a subgame-perfect Nash equilibrium is that the latter requires an additional assumption, which is called sequential rationality of the players. We refer, e.g., to [11] for more details on the previous definitions.

$$
\begin{cases}
\left(p_A^{M,C}\right)^{(\text{s.p.Nash},NPT)} = \frac{\gamma a}{2b}\,, \\
\left(p_A^{D,C}\right)^{(\text{s.p.Nash},NPT)} = (\text{not uniquely determined, but irrelevant as} \\
\qquad \left(q_A^{D,C}\right)^{(\text{s.p.Nash},NPT)} = 0)\,, \\
\left(p_B^{M,D}\right)^{(\text{s.p.Nash},NPT)} = \frac{a}{2b}\,, \\
\left(p_B^{D,C}\right)^{(\text{s.p.Nash},NPT)} = \frac{3a}{4b}\,, \\
\left(q_A^{M,C}\right)^{(\text{s.p.Nash},NPT)} = \frac{\gamma a}{2}\,, \\
\left(q_A^{D,C}\right)^{(\text{s.p.Nash},NPT)} = 0\,, \\
\left(q_B^{M,D}\right)^{(\text{s.p.Nash},NPT)} = \frac{a}{4}\,, \\
\left(q_B^{D,C}\right)^{(\text{s.p.Nash},NPT)} = \frac{a}{4}\,.
\end{cases}
\tag{68}
$$

(ii) **Second noncooperative game-theoretic model: parallel trade is permitted, but no parallel trade occurs at equilibrium (parallel trade threat).** Again, the interaction of the players (the manufacturer and the distributor) is described by a dynamic noncooperative game with perfect and complete information. In the first stage, the manufacturer sets the wholesale price $p_B^{M,D}$ for the distributor. Then, in the second stage, the distributor sets the retail price $p_B^{D,C}$ for the consumers in the country B. In the third stage, the manufacturer and the distributor choose simultaneously the prices $p_A^{M,C}$ and $p_A^{D,C}$ at which they sell the product to the consumers in the country A, according to a Bertrand duopoly model. No transfer payment is paid by the distributor to the manufacturer. Again, the game is solved in [12] by backward induction. The result of the equilibrium analysis (also reported in [7]) depends on the value of the per-unit parallel trade cost t. More precisely, two thresholds for t are defined in [7]:

$$
t_l = \begin{cases} \frac{a}{2b}\left(\gamma - \frac{5}{2}\right) & \text{if } \gamma \geq \frac{5}{2}\,, \\ 0 & \text{otherwise}\,, \end{cases} \tag{69}
$$

$$
t_h = \frac{a}{2b}(\gamma - 1)\,. \tag{70}
$$

Of course, $t_l \leq t_h$. Hence, one distinguishes among low values for t ($0 \leq t < t_l$), intermediate values for t ($t_l \leq t \leq t_h$), and high values for t ($t \geq t_h$). Notice that the first case is meaningful only when $t_l > 0$. Likewise in [7], in the following we use the symbols l, i, and h to denote the three respective cases. For each t, the following prices and quantities are obtained at a subgame-perfect Nash equilibrium (here, we use the superscript "PTT" to recall that the present game-theoretic model refers to parallel trade threat):

$$\text{if } 0 \leq t < t_l: \begin{cases} \left(p_A^{M,C}\right)^{(\text{s.p.Nash},l,PTT)} &= \frac{a}{6b}(2\gamma+1) + \frac{t}{3}\,, \\ \left(p_A^{D,C}\right)^{(\text{s.p.Nash},l,PTT)} &= \Big(\text{not uniquely determined, but irrelevant as} \\ & \qquad \left(q_A^{D,C}\right)^{(\text{s.p.Nash},l,PTT)} = 0\Big)\,, \\ \left(p_B^{M,D}\right)^{(\text{s.p.Nash},l,PTT)} &= \Big(\text{not uniquely determined, but irrelevant as} \\ & \qquad \left(q_B^{M,D}\right)^{(\text{s.p.Nash},l,PTT)} = 0\Big)\,, \\ \left(p_B^{D,C}\right)^{(\text{s.p.Nash},l,PTT)} &= \Big(\text{not uniquely determined, but irrelevant as} \\ & \qquad \left(q_B^{D,C}\right)^{(\text{s.p.Nash},l,PTT)} = 0\Big)\,, \\ \left(q_A^{M,C}\right)^{(\text{s.p.Nash},l,PTT)} &= \frac{a}{6}(4\gamma-1) - \frac{bt}{3}\,, \\ \left(q_A^{D,C}\right)^{(\text{s.p.Nash},l,PTT)} &= 0\,, \\ \left(q_B^{M,D}\right)^{(\text{s.p.Nash},l,PTT)} &= 0\,, \\ \left(q_B^{D,C}\right)^{(\text{s.p.Nash},l,PTT)} &= 0\,, \end{cases} \tag{71}$$

$$\text{if } t_l \leq t \leq t_h: \begin{cases} \left(p_A^{M,C}\right)^{(\text{s.p.Nash},i,PTT)} &= \frac{a}{6b}(2\gamma+1) + \frac{t}{3}\,, \\ \left(p_A^{D,C}\right)^{(\text{s.p.Nash},i,PTT)} &= \Big(\text{not uniquely determined, but irrelevant as} \\ & \qquad \left(q_A^{D,C}\right)^{(\text{s.p.Nash},i,PTT)} = 0\Big)\,, \\ \left(p_B^{M,D}\right)^{(\text{s.p.Nash},i,PTT)} &= \frac{a}{6b}(2\gamma+1) - \frac{2t}{3}\,, \\ \left(p_B^{D,C}\right)^{(\text{s.p.Nash},i,PTT)} &= \frac{a}{12b}(2\gamma+7) - \frac{t}{3}\,, \\ \left(q_A^{M,C}\right)^{(\text{s.p.Nash},i,PTT)} &= \frac{a}{6}(4\gamma-1) - \frac{bt}{3}\,, \\ \left(q_A^{D,C}\right)^{(\text{s.p.Nash},i,PTT)} &= 0\,, \\ \left(q_B^{M,D}\right)^{(\text{s.p.Nash},i,PTT)} &= \frac{a}{12}(5-2\gamma) + \frac{bt}{3}\,, \\ \left(q_B^{D,C}\right)^{(\text{s.p.Nash},i,PTT)} &= \frac{a}{12}(5-2\gamma) + \frac{bt}{3}\,, \end{cases} \tag{72}$$

$$
\text{if } t > t_h : \begin{cases}
\left(p_A^{M,C}\right)^{(\text{s.p.Nash},h,PTT)} &= \frac{\gamma a}{2b}\,, \\
\left(p_A^{D,C}\right)^{(\text{s.p.Nash},h,PTT)} &= (\text{not uniquely determined, but irrelevant as} \\
& \qquad \left(q_A^{D,C}\right)^{(\text{s.p.Nash},h,PTT)} = 0)\,, \\
\left(p_B^{M,D}\right)^{(\text{s.p.Nash},h,PTT)} &= \frac{a}{2b}\,, \\
\left(p_B^{D,C}\right)^{(\text{s.p.Nash},h,PTT)} &= \frac{3a}{4b}\,, \\
\left(q_A^{M,C}\right)^{(\text{s.p.Nash},h,PTT)} &= \frac{\gamma a}{2}\,, \\
\left(q_A^{D,C}\right)^{(\text{s.p.Nash},h,PTT)} &= 0\,, \\
\left(q_B^{M,D}\right)^{(\text{s.p.Nash},h,PTT)} &= \frac{a}{4}\,, \\
\left(q_B^{D,C}\right)^{(\text{s.p.Nash},h,PTT)} &= \frac{a}{4}\,.
\end{cases} \tag{73}
$$

One can notice that, for each of these subgame-perfect Nash equilibria, parallel trade actually does not occur. However, parallel trade freedom has an influence on the equilibrium behavior of the players (as one can see by comparing such equilibria with the ones obtained when parallel trade is forbidden; see the previous game-theoretic model).

Third noncooperative game-theoretic model: parallel trade is permitted, and it occurs at equilibrium. Also in this case, the interaction of the players (the manufacturer and the distributor) is described by a dynamic noncooperative game with perfect and complete information. In the first stage, the manufacturer sets the wholesale price $p_B^{M,D}$ for the distributor, together with a transfer payment $T_P \geq 0$, chosen inside the set of transfer payments that guarantee a non-negative surplus for the distributor (such a subset is determined in later stages). Then, in the second stage, the manufacturer and the distributor decide simultaneously the quantities $q_A^{M,C}$ and $q_A^{D,C}$ to be sold, respectively, to the consumers in the country A. At the same time, the distributor also decides the quantity $q_B^{D,C}$ to be sold to the consumers in the country B. The price $p_A^{M,C} = p_A^{D,C}$ is determined by a Cournot duopoly model, whereas the price $p_B^{D,C}$ is determined between by equating the offer from the distributor and the demand of the consumbers in the country B. Also this game is solved in [5] by backward induction. Again, the result of the equilibrium analysis depends on the value of the per-unit parallel trade cost t. In the following, we report[5] from [5] the prices and quantities that are obtained at a subgame-perfect Nash equilibrium for which parallel trade actually occurs (here, we use the superscript "PTO" to recall that the present game-theoretic model refers to the parallel trade occurrence at equilibrium). The following expressions hold under the assumptions $1 < \gamma \leq 2$ and $bt < \frac{3\gamma a}{14}$:

$$\begin{cases} \left(p_A^{M,C}\right)^{(\text{s.p.Nash},PTO)} = \dfrac{5\gamma a + 7bt}{13b}, \\ \left(p_A^{D,C}\right)^{(\text{s.p.Nash},PTO)} = \dfrac{5\gamma a + 7bt}{13b}, \\ \left(p_B^{M,D}\right)^{(\text{s.p.Nash},PTO)} = \dfrac{2\gamma a + 8bt}{13b}, \\ \left(p_B^{D,C}\right)^{(\text{s.p.Nash},PTO)} = \dfrac{a}{2b} + \dfrac{\gamma a + 4bt}{13b}, \\ \left(q_A^{M,C}\right)^{(\text{s.p.Nash},PTO)} = \dfrac{5\gamma a + 7bt}{13}, \\ \left(q_A^{D,C}\right)^{(\text{s.p.Nash},PTO)} = \dfrac{3\gamma a - 14bt}{13}, \\ \left(q_B^{M,D}\right)^{(\text{s.p.Nash},PTO)} = \dfrac{a}{2} + \dfrac{2\gamma a - 18bt}{13}, \\ \left(q_B^{D,C}\right)^{(\text{s.p.Nash},PTO)} = \dfrac{a}{2} - \dfrac{\gamma a + 4bt}{13}, . \end{cases} \tag{74}$$

The associated transfer payment is

$$(T_P)^{(\text{s.p.Nash},PTO)} = \frac{(9\gamma a - 42bt)^2}{1521b} + \frac{((13-2\gamma)a - 8bt)^2}{676b}. \tag{75}$$

Remark 4.1. All the equilibria above refer to the case in which the manufacturer decides to do research and developments, hence, it incurs the total fixed cost C_F. Moreover, one can see straightforwardly that all such equilibria are associated with non-negative surpluses of the distributor and of the consumers in the two countries, whereas the surplus of the manufacturer can be negative if the total fixed cost C_F is too large (but it is always non-negative for $C_F = 0$). However, due to their subgame-perfectness, the equilibria above change only slightly if one adds to the previous game-theoretic models the constraint that the surplus of the manufacturer has to be non-negative, making the manufacturer decide not to do

[5] As already mentioned, the model presented in [5] refers to the choices $\gamma = 1$ and $b = 1$. For uniformity of notation with the previous models, we have re-done all its computations removing the two assumptions $\gamma = 1$ and $b = 1$, obtaining the results shown in formula (74).

research and development in case its surplus is negative when research and development are done. By adding such a constraint, indeed, one obtains exactly the same expressions of the prices and quantities at equilibrium when the associated surplus of the manufacturer is non-negative, whereas all the quantities and surpluses become 0 otherwise. Finally, with these modifications the values of the global welfare associated with such equilibria are always non-negative, for each model of the global welfare function. ■

5. An application of the price of anarchy to parallel trade freedom/banning

In this section, for each of the global welfare models considered in Section 3 and each of the three noncooperative games described in Section 4, we apply an adaptation to our context of the concept of price of anarchy from [9], in order to compute the loss in efficiency in the optimization of the global welfare function, which is incurred when moving from the optimal solution determined by the global planner to the prices and quantities at the "worst" subgame-perfect Nash equilibrium of the game.

The following definition formalizes our adaptation of the definition of price of anarchy (PoA) from [9] to the three noncooperative games above. For uniformity of notation, in the following we consider the case in which the global planner can optimize also the transfer price (see, e.g., the optimization problems (45), (55), (66), and (61)). In the following definitions, in order to have non-negative values for the global welfare function in all ratios, we assume that both the global planner and the manufacturer optimize their strategies according to Remarks 3.2 and 4.1, respectively, i.e., taking into account the possibility not to do research and development.

Definition 5.1. *For each of the global welfare models considered in Section 3 and each of the three dynamic noncooperative games described in Section 4 modeling parallel trade banning/freedom, the price of anarchy (PoA) is defined as the ratio between the optimal value of the global welfare obtained by the hypothetical global planner of Section 3 under the same conditions of parallel trade banning/freedom and the assumption that the global planner can optimize also the transfer price, and the value of the global welfare associated with the "worst" subgame-perfect Nash equilibrium of the game (i.e., the one associated with the smallest value of the global welfare).*

Remark 5.2. Definition 5.1 differs from the one given in [9] for generic noncooperative games for the two following reasons, which are needed to adapt the original definition of price of anarchy to our context.

(i) In Definition 5.1, the numerator refers to the global planner, whereas the denominator refers to the worst equilibrium, whereas [9] does the opposite in its definition. This change in the definition is due to the fact that here we are considering the maximization of the global welfare, whereas [9] refers to a cost minimization problem. With this modification, one obtains in the present context a value of the price of anarchy that is always[6] greater than or equal to 1, likewise in the definition given in [9] for cost minimization problems.

(ii) In our context, we consider subgame-perfect Nash equilibria, instead than simply Nash equilibria. Indeed, all the equilibria reported in Section 4 are subgame-perfect Nash equilibria. ■

The importance of the concept of price of anarchy (in both Definition 5.1 and its original version stated in [9]) derives from the observation that it allows one to compare different noncooperative game-theoretic formulations, detecting when a change in the rules of the game (due, e.g., to the possible intervention of a policymaker) is needed to have a much more efficient (worst-case) equilibrium.

In the following, we also introduce a "normalized" price of anarchy, to better compare the equilibria of the two games for which parallel trade is, respectively, permitted/forbidden.

Definition 5.3. *For each of the global welfare models considered in Section 3 and each of the three dynamic noncooperative games described in Section 4 modeling parallel trade banning/freedom, the normalized price of anarchy ($PoA_{\text{norm.}}$) is defined as the ratio between the maximum of the optimal values of the global welfare obtained by the hypothetical global planner of Section 3 under each of the two conditions of parallel trade banning/freedom and the assumption that the global planner can optimize also the transfer price, and the value of the global welfare associated with the "worst" subgame-perfect Nash equilibrium of the game.*

[6]When the ratio in the definition of the price of anarchy (or normalized price of anarchy, see Definition 5.3) has the indeterminate form $\frac{0}{0}$, by convention we assign to it the value 1, as the numerator and the denominator are equal.

Remark 5.4. Since the optimization problems solved by the global planner when parallel trade is forbidden are more constrained than the ones for which parallel trade is permitted, the maximum in Definition 5.3 is always achieved in the situation in which there is parallel trade freedom. So, the two definitions of price of anarchy and normalized price of anarchy actually coincide for the games modeling parallel trade freedom. However, we have stated Definition 5.3 without any explicit reference to this fact, in order to obtain a definition that is more easily generalizable to other games. ■

Remark 5.5. As it will be shown in the next subsections, for each fixed choice of the global welfare model and of one the three noncooperative games, all the subgame-perfect Nash equilibria have the same value of the global welfare (which depends only on a, b, the heterogeneity parameter γ, and the per-unit parallel trade cost t), so there is no need for searching for the "worst" equilibrium, as all such equilibria are equivalent in efficiency. ■

In the following, we express the price of anarchy/normalized price of anarchy for the various models of the global welfare functions and noncooperative games considered in the paper. In order to simplify the presentation, all the expressions of the price of anarchy/normalized price of anarchy contained in the following Propositions 5.6, 5.8, and 5.10, refer to the case $C_F = 0$, for which both the global planner and the manufacturer decide to do research and development, and for which both the numerator and the denominator in the definitions of the price of anarchy/normalized price of anarchy have simplified forms. The extension to the case $C_F > 0$ can be obtained straightforwardly, using the more general expressions for the numerator and denominator presented in Sections 3 and 4.

5.1. *Evaluation of the prices of anarchy under the Bentham global welfare model*

The following proposition provides expressions for the prices/normalized prices of anarchy associated with the Bentham global welfare model and the three games considered in Section 4, when $C_F = 0$.

Proposition 5.6. *(i) For the case of the Bentham global welfare model and the game modeling parallel trade banning, the price of anarchy coincides with the normalized price of anarchy, and has the following expression for $C_F = 0$:*

$$PoA_{NPT}^{(B)}(\gamma) = PoA_{NPT,\mathrm{norm.}}^{(B)}(\gamma) = \frac{\frac{(\gamma^2+1)a^2}{2b}}{\frac{a^2}{32b}(12\gamma^2+7)} = \frac{16(\gamma^2+1)}{12\gamma^2+7}\,. \quad (76)$$

(ii) For the case of the Bentham global welfare model and the game modeling parallel trade threat, the price of anarchy coincides with the normalized price of anarchy, and has the following expression for $C_F = 0$:

$$PoA^{(B)}_{PTT}(\gamma,t) = PoA^{(B)}_{PTT,\text{norm.}}(\gamma,t) = \begin{cases} \dfrac{\frac{(\gamma^2+1)a^2}{2b}}{\frac{a^2}{72b}(32\gamma^2-4\gamma-1)-\frac{1}{18}(bt^2+at+2\gamma at)} & \text{if } 0 \le t < t_l\,, \\ \dfrac{\frac{(\gamma^2+1)a^2}{2b}}{\frac{a^2}{288b}(124\gamma^2-44\gamma+91)-\frac{1}{36}(4bt^2+2\gamma at-5at)} & \text{if } t_l \le t \le t_h\,, \\ \dfrac{\frac{(\gamma^2+1)a^2}{2b}}{\frac{a^2}{32b}(12\gamma^2+7)} = \dfrac{16(\gamma^2+1)}{12\gamma^2+7} & \text{if } t > t_h\,. \end{cases} \tag{77}$$

(iii) For the case of the Bentham global welfare model and the game modeling parallel trade occurrence, the price of anarchy coincides with the normalized price of anarchy, and has the following expression for $C_F = 0$, $1 < \gamma \le 2$, and $0 \le t < \frac{3\gamma a}{14b}$:

$$\begin{aligned} PoA^{(B)}_{PTO}(\gamma,t) = PoA^{(B)}_{PTO,\text{norm.}}(\gamma,t) &= \frac{\frac{(\gamma^2+1)a^2}{2b}}{\frac{\frac{11}{26}(\gamma a)^2-\frac{6}{13}(\gamma a)(bt)+\frac{23}{26}(bt)^2+\frac{3}{8}a^2-\frac{1}{26}(\gamma a)a-\frac{2}{13}bt}{b}} \\ &= \frac{(\gamma^2+1)a^2}{\frac{11}{13}(\gamma a)^2-\frac{12}{13}(\gamma a)(bt)+\frac{23}{13}(bt)^2+\frac{3}{4}a^2-\frac{1}{13}(\gamma a)a-\frac{4}{13}a(bt)}\,. \end{aligned} \tag{78}$$

Proof. (i) As reported in [7] (see also [12] for a derivation), for the Bentham model and the game modeling parallel trade banning, the value of the global welfare associated with the subgame-perfect Nash equilibrium provided by formula (68) has the following expression[7] for $C_F = 0$:

$$GW^{(B,\text{s.p.Nash},NPT)}_{NPT}(\gamma) = \frac{a^2}{32b}(12\gamma^2+7)\,. \tag{79}$$

Then, (77) is derived by applying (79), Propositions 3.3 and 3.5, and Definitions 5.1 and 5.3.

(ii) As reported in [7] (see also [12] for a derivation), for the Bentham global welfare model and the game modeling parallel trade threat, the values of the global welfare associated with the subgame-perfect Nash equilibria provided by formulas (71), (72), and (73) have the following expressions[8] for $C_F = 0$:

[7] Such an expression can also be derived by using formulas (17) and (68).

[8] Such expressions can also be derived by using formulas (16), (71), (72), and (73).

$$
\begin{aligned}
GW_{PT}^{(B,\text{s.p.Nash},l,PTT)}(\gamma,t) &= \frac{a^2}{72b}(32\gamma^2-4\gamma-1)-\frac{1}{18}(bt^2+at+2\gamma at)\,,\\
GW_{PT}^{(B,\text{s.p.Nash},i,PTT)}(\gamma,t) &= \frac{a^2}{288b}(124\gamma^2-44\gamma+91)-\frac{1}{36}(4bt^2+2\gamma at-5at)\,,\\
GW_{PT}^{(B,\text{s.p.Nash},h,PTT)}(\gamma,t) &= \frac{a^2}{32b}(12\gamma^2+7)\,.
\end{aligned}
\tag{80}
$$

Then, (77) is derived by applying (80), Proposition 3.3, Definitions 5.1 and 5.3, and Remark 5.4.

(iii) As reported in [5], for the Bentham global welfare model and the game modeling parallel trade occurrence, the value of the global welfare associated with the subgame-perfect Nash equilibrium provided by formula (74) has the following expression[9] for $C_F = 0$, $1 < \gamma \leq 2$, and $0 \leq t < \frac{3\gamma a}{14b}$:

$$
\begin{aligned}
GW_{PT}^{(B,\text{s.p.Nash},PTO)}(\gamma,t) &= \frac{\left(\gamma a+\frac{2\gamma a+8bt}{13}+bt\right)^2}{9b}+\frac{2\gamma a+8bt}{13b}\,\frac{\gamma a-2\frac{2\gamma a+8bt}{13}-2bt}{3}\\
&+\frac{\left(\gamma a-2\frac{2\gamma a+8bt}{13}-2bt\right)^2}{9b}+\frac{a^2-\left(\frac{2\gamma a+8bt}{13}\right)^2}{4b}\\
&+\frac{\left(\gamma a-\frac{5\gamma a+7bt}{13}\right)^2}{2b}+\frac{\left(\frac{a}{2}-\frac{\gamma a+4bt}{13}\right)^2}{2b}\\
&=\frac{\frac{11}{26}(\gamma a)^2-\frac{6}{13}(\gamma a)(bt)+\frac{23}{26}(bt)^2+\frac{3}{8}a^2-\frac{1}{26}(\gamma a)a-\frac{2}{13}a(bt)}{b}\,.
\end{aligned}
\tag{81}
$$

Then, (78) is derived by applying (81), Proposition 3.3, Definitions 5.1 and 5.3, and Remark 5.4. ■

Remark 5.7. By using formula (76), one can also see that $PoA_{NPT}^{(B)}(\gamma)$ is a decreasing function of $\gamma \in (1,+\infty)$, $\lim_{\gamma\to 1^+} PoA_{NPT}^{(B)}(\gamma) = \frac{32}{19}$, and

$$
\lim_{\gamma\to+\infty} PoA_{NPT}^{(B)}(\gamma) = \frac{4}{3}\,. \tag{82}
$$

[9]Such an expression can also be derived by using formulas (16) and (74).

Similarly, by exploiting formula (77), one can see that

$$\lim_{\gamma \to 1^+} PoA^{(B)}_{PTT}(\gamma, t) = \begin{cases} \dfrac{32}{19} & \text{if } t = 0\,, \\ \dfrac{32}{19} & \text{if } t > 0\,, \end{cases} \tag{83}$$

(since, for γ sufficiently close to 1, one gets $t_l = 0$, and $t_h \simeq 0$), and that, for each fixed $t \geq 0$,

$$\lim_{\gamma \to +\infty} PoA^{(B)}_{PTT}(\gamma, t) = \frac{9}{8}\,. \tag{84}$$

Finally, by exploiting formula (78), one can see that, for $0 \leq t < \frac{3\gamma a}{14b}$,

$$\lim_{\gamma \to 1^+} PoA^{(B)}_{PTO}(\gamma, t) = \frac{1}{\frac{79}{104} - \frac{8}{13}\left(\frac{bt}{a}\right) + \frac{23}{26}\left(\frac{bt}{a}\right)^2}\,, \tag{85}$$

and

$$PoA^{(B)}_{PTO}(2, t) = \frac{1}{\frac{249}{13} - \frac{64}{65}\left(\frac{bt}{a}\right) + \frac{23}{65}\left(\frac{bt}{a}\right)^2}\,. \tag{86}$$

■

5.2. Evaluation of the prices of anarchy under the first specification of the Rawls global welfare model

The following proposition provides expressions for the prices/normalized prices of anarchy associated with the first specification of the Rawls global welfare model and the three games considered in Section 4, when $C_F = 0$.

Proposition 5.8. *(i) For the case of the first specification of the Rawls global welfare model and the game modeling parallel trade banning, the price of anarchy has the following expression for $C_F = 0$:*

$$PoA^{(R,I)}_{NPT}(\gamma) = \frac{\frac{a^2}{2b}}{\frac{3a^2}{32b}} = \frac{16}{3}\,, \tag{87}$$

whereas the normalized price of anarchy is

$$PoA^{(R,I)}_{NPT,\text{norm.}}(\gamma) = \frac{\frac{(\gamma^2+1)a^2}{4b}}{\frac{3a^2}{32b}} = \frac{8(\gamma^2+1)}{3}\,. \tag{88}$$

(ii) For the case of the first specification of the Rawls global welfare model and the game modeling parallel trade threat, the price of anarchy coincides with the normalized price of anarchy, and has the following expression for $C_F = 0$:

$$PoA^{(R,I)}_{PTT}(\gamma,t) = PoA^{(R,I)}_{PTT,\text{norm.}}(\gamma,t) = \begin{cases} +\infty & \text{if } 0 \leq t < t_l\,, \\ \dfrac{\frac{(\gamma^2+1)a^2}{4b}}{GW^{(R,I,\text{s.p.Nash},i,PT)}_{PTT}(\gamma,t)} & \text{if } t_l \leq t \leq t_h\,, \\ \dfrac{\frac{(\gamma^2+1)a^2}{4b}}{\frac{3a^2}{32b}} = \dfrac{8(\gamma^2+1)}{3} & \text{if } t > t_h\,, \end{cases} \tag{89}$$

where $GW^{(R,I,\text{s.p.Nash},i,PT)}_{PTT}(\gamma,t)$ is the value of the first specification of the Rawls global welfare model computed when using the prices and quantities associated with the subgame-perfect Nash equilibrium (72)[10].

(iii) For the case of the first specification of the Rawls global welfare model and the game modeling parallel trade occurrence, the price of anarchy coincides with the normalized price of anarchy, and has the following expression for $C_F = 0$, $1 < \gamma \leq 2$, and $0 \leq t < \frac{3\gamma a}{14b}$:

$$PoA^{(R,I)}_{PTO}(\gamma,t) = PoA^{(R,I)}_{PTO,\text{norm.}}(\gamma,t) = \frac{\frac{(\gamma^2+1)a^2}{4b}}{\frac{\left(\frac{a}{2}-\frac{\gamma a+4bt}{13}\right)^2}{2b}} = \frac{(\gamma^2+1)a^2}{2\left(\frac{a}{2}-\frac{\gamma a+4bt}{13}\right)^2}\,. \tag{90}$$

Proof. (i) For the first specification of the Rawls global welfare model and the game modeling parallel trade banning, the value of the global welfare associated with the subgame-perfect Nash equilibrium provided by formula (68) has the fol-

[10] See formula (93) in the proof for its expression.

lowing expression for $C_F = 0$, which can be derived by using also formula (19):

$$\begin{aligned} GW_{NPT}^{(R,I,\text{s.p.Nash},NPT)}(\gamma) &= \min\left\{ \frac{a}{2b}\frac{a}{4} + \frac{\gamma a}{b}\frac{\gamma a}{2} - \frac{\left(\frac{\gamma a}{2}\right)^2}{2b}, \right. \\ &\qquad \left. -\frac{a}{2b}\frac{a}{4} + \frac{a}{b}\frac{a}{4} - \frac{\left(\frac{a}{4}\right)^2}{2b} \right\} \\ &= \min\left\{ \frac{(3\gamma^2+1)a^2}{8b}, \frac{3a^2}{32b} \right\} \\ &= \frac{3a^2}{32b}, \end{aligned} \tag{91}$$

where the mimimum is clearly achieved in correspondence of the country B. Then, (87) and (88) are derived by applying (91), Propositions 3.6 and 3.7, and Definitions 5.1 and 5.3.

(ii) For the first specification of the Rawls global welfare model and the game modeling parallel trade threat, the values of the global welfare associated with the subgame-perfect Nash equilibria provided by formulas (71), (72), and (73) have the following expressions for $C_F = 0$, which can be derived by using also formula (18):

$$\begin{aligned} GW_{PTT}^{(R,I,\text{s.p.Nash},l,PT)}(\gamma,t) &= \min\left\{ \frac{\gamma a}{b}\left(\frac{a}{6}(4\gamma-1) - \frac{bt}{3}\right) - \frac{\left(\frac{a}{6}(4\gamma-1) - \frac{bt}{3}\right)^2}{2b}, \right. \\ &\qquad \left. 0 \right\} \\ &= 0, \end{aligned} \tag{92}$$

$$
\begin{aligned}
GW_{PTT}^{(R,I,\text{s.p.Nash},i,PT)}(\gamma,t) &= \min\left\{\left(\frac{a}{6b}(2\gamma+1)-\frac{2t}{3}\right)\left(\frac{a}{12}(5-2\gamma)+\frac{bt}{3}\right)\right. \\
&\quad +\frac{\gamma a}{b}\left(\frac{a}{6}(4\gamma-1)-\frac{bt}{3}\right)-\frac{\left(\frac{a}{6}(4\gamma-1)-\frac{bt}{3}\right)^2}{2b}, \\
&\quad \left(\frac{a}{b}-\left(\frac{a}{6b}(2\gamma+1)-\frac{2t}{3}\right)\right)\left(\frac{a}{12}(5-2\gamma)+\frac{bt}{3}\right) \\
&\quad \left.-\frac{\left(\frac{a}{12}(5-2\gamma)+\frac{bt}{3}\right)^2}{2b}\right\} \\
&= \left(\frac{a}{b}-\left(\frac{a}{6b}(2\gamma+1)-\frac{2t}{3}\right)\right)\left(\frac{a}{12}(5-2\gamma)+\frac{bt}{3}\right) \\
&\quad -\frac{\left(\frac{a}{12}(5-2\gamma)+\frac{bt}{3}\right)^2}{2b}, \qquad (93)
\end{aligned}
$$

$$
\begin{aligned}
GW_{PTT}^{(R,I,\text{s.p.Nash},h,PT)}(\gamma,t) &= \min\left\{\frac{a}{2b}\frac{a}{4}+\frac{\gamma a}{b}\frac{\gamma a}{2}-\frac{\left(\frac{\gamma a}{2}\right)^2}{2b},\right. \\
&\quad \left.-\frac{a}{2b}\frac{a}{4}+\frac{a}{b}\frac{a}{4}-\frac{\left(\frac{a}{4}\right)^2}{2b}\right\} \\
&= \min\left\{\frac{(3\gamma^2+1)a^2}{8b},\frac{3a^2}{32b}\right\} \\
&= \frac{3a^2}{32b}, \qquad (94)
\end{aligned}
$$

where all the minima above are achieved in correspondence of the country B[11]. Then, (89) is derived by applying (92), (93), (94), Proposition 3.6, Definitions 5.1 and 5.3, and Remark 5.4.

[11]For the cases of formulas (92) and (94), the proof that the minima are achieved in correspondence of the country B is immediate; for the case of formula (93), this is proved by observing that the national welfare of the country A is greater than or equal to the surplus of the consumers of the country A obtained when the wholesale price $p_A^{M,C}$ is equal to 0, and the national welfare of the country A is smaller than or equal to the surplus of the consumers of the country B obtained when the retail price $p_A^{M,C}$ is equal to 0. Now, as $\gamma > 1$ and

(iii) For the first specification of the Rawls global welfare model and the game modeling parallel trade threat, the value of the global welfare associated with the subgame-perfect Nash equilibrium provided by formula (74) has the following expression for $C_F = 0$, $1 < \gamma \leq 2$, and $0 \leq t < \frac{3\gamma a}{14b}$, which can be derived by using also formula (18):

$$\begin{aligned} GW_{PTT}^{(R,I,\text{s.p.Nash},PT)}(\gamma,t) &= \min\left\{ \frac{\left(\gamma a + \frac{2\gamma a+8bt}{13} + bt\right)^2}{9b} + \frac{2\gamma a + 8bt}{13b}\frac{\gamma a - 2\frac{2\gamma a+8bt}{13} - 2bt}{3} \right. \\ &\quad + \frac{\left(\gamma a - 2\frac{2\gamma a+8bt}{13} - 2bt\right)^2}{9b} + \frac{a^2 - \left(\frac{2\gamma a+8bt}{13}\right)^2}{4b} \\ &\quad + \frac{\left(\gamma a - \frac{5\gamma a+7bt}{13}\right)^2}{2b}, \\ &\quad \left. \frac{\left(\frac{a}{2} - \frac{\gamma a+4bt}{13}\right)^2}{2b} \right\} \\ &= \frac{\left(\frac{a}{2} - \frac{\gamma a+4bt}{13}\right)^2}{2b}, \end{aligned} \tag{95}$$

where the minimum above is achieved in correspondence of the country B, as it can be shown by simple algebraic manipulations. Then, (90) is derived by applying (95), Proposition 3.6, Definitions 5.1 and 5.3, and Remark 5.4. ■

Remark 5.9. By using formula (87), one can see that $PoA_{NPT}^{(R,I)}(\gamma)$ is a constant function of $\gamma \in (1, +\infty)$, so $\lim_{\gamma \to 1^+} PoA_{NPT}^{(R,I)}(\gamma) = \frac{16}{3}$, and

$$\lim_{\gamma \to +\infty} PoA_{NPT}^{(R,I)}(\gamma) = \frac{16}{3}. \tag{96}$$

Finally, by exploiting formula (88), one can see that $PoA_{NPT,\text{norm.}}^{(R,I)}(\gamma)$ is an increasing function of $\gamma \in (1, +\infty)$, $\lim_{\gamma \to 1^+} PoA_{NPT,\text{norm.}}^{(R,I)}(\gamma) = \frac{16}{3}$, and

$$\lim_{\gamma \to +\infty} PoA_{NPT,\text{norm.}}^{(R,I)}(\gamma) = +\infty. \tag{97}$$

$\left(q_A^{M,C}\right)^{(\text{s.p.Nash},i,PT)} \geq \left(q_B^{D,C}\right)^{(\text{s.p.Nash},i,PT)}$, the first surplus is greater than or equal to the second one, hence the minimum in formula (93) is achieved in correspondence of the country B.

Similarly, by exploiting formulas (89) and (93), one can see that

$$\lim_{\gamma \to 1^+} PoA_{PTT}^{(R,I)}(\gamma, t) = \begin{cases} \dfrac{16}{3} & \text{if } t = 0\,, \\ \dfrac{16}{3} & \text{if } t > 0\,, \end{cases} \tag{98}$$

(since, for γ sufficiently close to 1, one gets $t_l = 0$, and $t_h \simeq 0$), and that, for each fixed $t \geq 0$,

$$\lim_{\gamma \to +\infty} PoA_{PTT}^{(R,I)}(\gamma, t) = +\infty\,. \tag{99}$$

Moreover, for each fixed $\gamma > 1$,

$$\lim_{t \to +\infty} PoA_{PTT}^{(R,I)}(\gamma, t) = \frac{8(\gamma^2 + 1)}{3}\,. \tag{100}$$

Finally, by using formula (90), one can see that, for $0 \leq t < \frac{3\gamma a}{14b}$,

$$\lim_{\gamma \to 1^+} PoA_{PTO}^{(R,I)}(\gamma, t) = \frac{1}{\left(\frac{1}{2} - \frac{1+4\frac{bt}{a}}{13}\right)^2}\,, \tag{101}$$

and

$$PoA_{PTO}^{(R,I)}(2, t) = \frac{5}{\left(\frac{1}{2} - \frac{4\left(1+\frac{bt}{a}\right)}{13}\right)^2}\,. \tag{102}$$

■

5.3. Evaluation of the prices of anarchy under the second specification of the Rawls global welfare model

Finally, the following proposition provides expressions for the prices/normalized prices of anarchy associated with the second specification of the Rawls global welfare model and the three games considered in Section 4, when $C_F = 0$.

Proposition 5.10. *(i) For the case of the second specification of the Rawls global welfare model and the game modeling parallel trade banning, the price of anarchy has the following expression for $C_F = 0$:*

$$PoA_{NPT}^{(R,II)}(\gamma) = \frac{\frac{a^2}{4b}}{\frac{a^2}{32b}} = 8\,, \tag{103}$$

whereas the normalized price of anarchy is

$$PoA_{NPT}^{(R,II)}(\gamma) = \begin{cases} \dfrac{\frac{(\gamma^2+1)a^2}{8b}}{\frac{a^2}{32b}} = 4(\gamma^2+1) & \text{if } 1 < \gamma \leq \sqrt{3}\,, \\ \dfrac{\frac{a^2}{2b}}{\frac{a^2}{32b}} = 16 & \text{if } \gamma > \sqrt{3}\,. \end{cases} \tag{104}$$

(ii) For the case of the second specification of the Rawls global welfare model and the game modeling parallel trade threat, the price of anarchy coincides with the normalized price of anarchy, and has the following expression for $C_F = 0$:

$$\begin{aligned} & PoA_{PTT}^{(R,II)}(\gamma,t) \\ = \;& PoA_{PTT,\text{norm.}}^{(R,II)}(\gamma,t) \\ = \;& \begin{cases} \dfrac{\frac{(\gamma^2+1)a^2}{8b}}{GW_{PTT}^{(R,II,\text{s.p.Nash},i,PT)}(\gamma,t)} & \text{if } 1 < \gamma \leq \sqrt{3} \text{ and } 0 \leq t \leq t_h\,, \\ \dfrac{\frac{(\gamma^2+1)a^2}{8b}}{\frac{a^2}{32b}} = 4(\gamma^2+1) & \text{if } 1 < \gamma \leq \sqrt{3} \text{ and } t > t_h\,, \\ \dfrac{\frac{a^2}{2b}}{GW_{PTT}^{(R,II,\text{s.p.Nash},i,PT)}(\gamma,t)} & \text{if } \sqrt{3} < \gamma \leq \dfrac{5}{2} \text{ and } 0 \leq t \leq t_h\,, \\ \dfrac{\frac{a^2}{2b}}{\frac{a^2}{32b}} = 16 & \text{if } \sqrt{3} < \gamma \leq \dfrac{5}{2} \text{ and } t > t_h\,, \\ +\infty & \text{if } \gamma > \dfrac{5}{2} \text{ and } 0 \leq t < t_l\,, \\ \dfrac{\frac{a^2}{2b}}{GW_{PTT}^{(R,II,\text{s.p.Nash},i,PT)}(\gamma,t)} & \text{if } \gamma > \dfrac{5}{2} \text{ and } t_l \leq t \leq t_h\,, \\ \dfrac{\frac{a^2}{2b}}{\frac{a^2}{32b}} = 16 & \text{if } \gamma > \dfrac{5}{2} \text{ and } t > t_h\,, \end{cases} \end{aligned} \tag{105}$$

where $GW_{PTT}^{(R,II,\text{s.p.Nash},i,PT)}(\gamma,t)$ is the value of the second specification of the Rawls global welfare model computed when using the prices and quantities associated with the subgame-perfect Nash equilibrium (72)[12].

[12] See formula (109) in the proof for its expression.

(iii) For the case of the second specification of the Rawls global welfare model and the game modeling parallel trade occurrence, the price of anarchy coincides with the normalized price of anarchy, and has the following expression for $C_F = 0$, $1 < \gamma \leq 2$, and $0 \leq t < \frac{3\gamma a}{14b}$:

$$PoA^{(R,II)}_{PTO}(\gamma, t) = PoA^{(R,II)}_{PTO,\text{norm.}}(\gamma, t) = +\infty \,. \tag{106}$$

Proof. (i) For the second specification of the Rawls global welfare model and the game modeling parallel trade banning, the value of the global welfare associated with the subgame-perfect Nash equilibrium provided by formula (68) has the following expression for $C_F = 0$, which can be derived by using also formula (21):

$$\begin{aligned} GW^{(R,II,\text{s.p.Nash},NPT)}_{NPT}(\gamma) &= \min\left\{ \frac{\gamma a}{2b}\frac{\gamma a}{2} + \frac{a}{2b}\frac{a}{4} = \frac{(2\gamma^2+1)a^2}{8b}, \right. \\ &\qquad \left(\frac{3a}{4b} - \frac{a}{2b}\right)\frac{a}{4} = \frac{a^2}{16b}, \\ &\qquad \frac{\gamma a}{b}\frac{\gamma a}{2} - \frac{\gamma a}{2b}\frac{\gamma a}{2} - \frac{\left(\frac{\gamma a}{2}\right)^2}{2b} = \frac{\gamma^2 a^2}{8b}, \\ &\qquad \left. \left(\frac{a}{b} - \frac{3a}{4b}\right)\frac{a}{4} - \frac{\left(\frac{a}{4}\right)^2}{2b} = \frac{a^2}{32b} \right\} \\ &= \frac{a^2}{32b}, \end{aligned} \tag{107}$$

where the mimimum is clearly achieved in correspondence of the consumers in the country B. Then, (103) and (104) are derived by applying (107), Propositions 3.8 and 3.9, and Definitions 5.1 and 5.3.
(ii) For the second specification of the Rawls global welfare model and the game modeling parallel trade threat, the values of the global welfare associated with the subgame-perfect Nash equilibria provided by formulas (71), (72), and (73) have the following expressions for $C_F = 0$, which can be derived by using also formula (20):

$$
\begin{aligned}
GW_{PT}^{(R,II,\text{s.p.Nash},l,PTT)}(\gamma,t) &= \min\Bigg\{\left(\frac{a}{6b}(2\gamma+1)+\frac{t}{3}\right)\left(\frac{a}{6}(4\gamma-1)-\frac{bt}{3}\right), \\
&\qquad 0\,, \\
&\qquad \frac{\gamma a}{b}\left(\frac{a}{6}(4\gamma-1)-\frac{bt}{3}\right) \\
&\qquad -\left(\frac{a}{6b}(2\gamma+1)+\frac{t}{3}\right)\left(\frac{a}{6}(4\gamma-1)-\frac{bt}{3}\right) \\
&\qquad -\frac{\left(\frac{a}{6}(4\gamma-1)-\frac{bt}{3}\right)^2}{2b}\,, \\
&\qquad 0\Bigg\} \\
&= 0\,,
\end{aligned} \tag{108}
$$

$$
\begin{aligned}
GW_{PT}^{(R,II,\text{s.p.Nash},i,PTT)}(\gamma,t) &= \min\Bigg\{\left(\frac{a}{6b}(2\gamma+1)+\frac{t}{3}\right)\left(\frac{a}{6}(4\gamma-1)-\frac{bt}{3}\right) \\
&\qquad +\left(\frac{a}{6b}(2\gamma+1)-\frac{2t}{3}\right)\left(\frac{a}{12}(5-2\gamma)+\frac{bt}{3}\right), \\
&\qquad \left(\frac{a}{12b}(2\gamma+7)-\frac{t}{3}-\left(\frac{a}{6b}(2\gamma+1)-\frac{2t}{3}\right)\right) \\
&\qquad \cdot\left(\frac{a}{12}(5-2\gamma)+\frac{bt}{3}\right), \\
&\qquad -\left(\frac{a}{6b}(2\gamma+1)+\frac{t}{3}\right)\left(\frac{a}{6}(4\gamma-1)-\frac{bt}{3}\right) \\
&\qquad +\frac{\gamma a}{b}\left(\frac{a}{6}(4\gamma-1)-\frac{bt}{3}\right)-\frac{\left(\frac{a}{6}(4\gamma-1)-\frac{bt}{3}\right)^2}{2b}\,, \\
&\qquad -\left(\frac{a}{12b}(2\gamma+7)-\frac{t}{3}\right)\left(\frac{a}{12}(5-2\gamma)+\frac{bt}{3}\right) \\
&\qquad +\frac{a}{b}\left(\frac{a}{12}(5-2\gamma)+\frac{bt}{3}\right)-\frac{\left(\frac{a}{12}(5-2\gamma)+\frac{bt}{3}\right)^2}{2b}\Bigg\}\,,
\end{aligned} \tag{109}
$$

$$
\begin{aligned}
GW_{PT}^{(R,II,\text{s.p.Nash},h,PTT)}(\gamma,t) &= \min\Bigg\{\frac{\gamma a}{2b}\frac{\gamma a}{2}+\frac{a}{2b}\frac{a}{4}=\frac{(2\gamma^2+1)a^2}{8b}\,, \\
&\qquad \left(\frac{3a}{4b}-\frac{a}{2b}\right)\frac{a}{4}=\frac{a^2}{16b}\,, \\
&\qquad \frac{\gamma a}{b}\frac{\gamma a}{2}-\frac{\gamma a}{2b}\frac{\gamma a}{2}-\frac{\left(\frac{\gamma a}{2}\right)^2}{2b}=\frac{\gamma^2 a^2}{8b}\,, \\
&\qquad \left(\frac{a}{b}-\frac{3a}{4b}\right)\frac{a}{4}-\frac{\left(\frac{a}{4}\right)^2}{2b}=\frac{a^2}{32b}\Bigg\} \\
&= \frac{a^2}{32b}\,,
\end{aligned} \tag{110}
$$

where the minima in (108) and (110) are achieved in correspondence of the consumers in the country B[13]. Then, (105) is derived by applying (108), (109), (110), Proposition 3.8, Definitions 5.1 and 5.3, and Remark 5.4.

(iii) For the second specification of the Rawls global welfare model and the game modeling parallel trade occurrence, the value of the global welfare associated with the subgame-perfect Nash equilibrium provided by formula (74) has the following expression for $C_F = 0$, $1 < \gamma \leq 2$, and $0 \leq t < \frac{3\gamma a}{14b}$, which can be derived by using also formula (20):

$$\begin{aligned} GW_{PT}^{(R,II,\text{s.p.Nash},PTO)}(\gamma,t) &= \min\Bigg\{ \frac{\left(\gamma a + \frac{2\gamma a+8bt}{13} + bt\right)^2}{9b} + \frac{2\gamma a + 8bt}{13b}\,\frac{\gamma a - 2\frac{2\gamma a+8bt}{13} - 2bt}{3} \\ &\quad + \frac{\left(\gamma a - 2\frac{2\gamma a+8bt}{13} - 2bt\right)^2}{9b} + \frac{a^2 - \left(\frac{2\gamma a+8bt}{13}\right)^2}{4b}, \\ &\quad 0, \\ &\quad \frac{\left(\gamma a - \frac{5\gamma a+7bt}{13}\right)^2}{2b}, \\ &\quad \frac{\left(\frac{a}{2} - \frac{\gamma a+4bt}{13}\right)^2}{2b} \Bigg\} \\ &= 0\,. \end{aligned} \tag{111}$$

where the minimum above is achieved in correspondence of the distributor (due to the presence of the transfer payment(75) at equilibrium). Then, (106) is derived by applying (111), Proposition 3.8, Definitions 5.1 and 5.3, and Remark 5.4. ■

Remark 5.11. By using formula (103), one can see that $PoA_{NPT}^{(R,II)}(\gamma)$ is a constant function of $\gamma \in (1, +\infty)$, so $\lim_{\gamma \to 1^+} PoA_{NPT}^{(R,II)}(\gamma) = 8$, and

$$\lim_{\gamma \to +\infty} PoA_{NPT}^{(R,II)}(\gamma) = 8\,. \tag{112}$$

Finally, by exploiting formula (104), one can see that $PoA_{NPT,\text{norm.}}^{(R,II)}(\gamma)$ is a non-

[13] An in-depth investigation of where the minimum is achieved in formula (109) by varying its parameters is beyond the scope of the work. However, for fixed values of the parameters, this can be simply determined by comparing the four expressions in (109).

decreasing function of $\gamma \in (1, +\infty)$, $\lim_{\gamma \to 1^+} PoA^{(R,II)}_{NPT,\text{norm.}}(\gamma) = 8$, and

$$\lim_{\gamma \to +\infty} PoA^{(R,II)}_{NPT,\text{norm.}}(\gamma) = 16\,. \tag{113}$$

Similarly, by exploiting formulas (105) and (109), one can see that

$$\lim_{\gamma \to 1^+} PoA^{(R,II)}_{PTT}(\gamma, t) = \begin{cases} 8 & \text{if } t = 0\,, \\ 8 & \text{if } t > 0\,, \end{cases} \tag{114}$$

(since, for γ sufficiently close to 1, one gets $t_l = 0$, and $t_h \simeq 0$), and that, for each fixed $t \geq 0$,

$$\lim_{\gamma \to +\infty} PoA^{(R,II)}_{PTT}(\gamma, t) = +\infty\,. \tag{115}$$

Moreover, for each fixed $\gamma > 1$,

$$\lim_{t \to +\infty} PoA^{(R,II)}_{PTT}(\gamma, t) = \begin{cases} 4(\gamma^2 + 1) & \text{if } 1 < \gamma \leq \sqrt{3}\,, \\ 16 & \text{if } \gamma > \sqrt{3}\,, \end{cases} \tag{116}$$

Finally, by using formula (106), one can see that, for $0 \leq t < \frac{3\gamma a}{14b}$, $PoA^{(R,II)}_{PTO}(\gamma, t) = +\infty$ for every $\gamma \in (1, 2]$. ■

5.4. A summary of the obtained results

Table 1 summarizes the results obtained about the price of anarchy/normalized price of anarchy for the noncooperative games and global welfare models considered in the paper. Limiting the comparison to the cases of parallel trade banning/parallel trade threat, which - differently from the case of parallel trade occurrence - do not impose restrictions on γ, an inspection ot Table 1 shows the following:

(i) for the Bentham global welfare model, using similar arguments as the ones used in the proofs of [7, Propositions 3-5], one can prove that, for some values of γ and t, the price of anarchy when parallel trade is forbidden is greater than the price of anarchy when there is parallel trade threat; however, there exist also some values of γ and t for which the opposite holds (see also the plots in Figures 2 and 3);

(ii) for the first specification of the Rawls global welfare model and $0 \leq t < t_l$ or $t > t_h$, the price of anarchy when parallel trade is forbidden is always smaller than the price of anarchy when there is parallel trade threat, since $\frac{16}{3} < \frac{8(\gamma^2+1)}{3} < +\infty$. As confirmed by numerical results (see also the plots in Figures 2 and 3), this is still true for $t_l \leq t \leq t_h$. However, when the normalized price of anarchy is considered, the two cases are equivalent for $t > t_h$;

(iii) for the second specification of the Rawls global welfare model and $\left(\gamma > \frac{5}{2}, 0 \leq t < t_l\right)$ or $t > t_h$, the price of anarchy when parallel trade is forbidden is always smaller than the price of anarchy when there is parallel trade threat, since $8 < 16 < 4(\gamma^2 + 1) < +\infty$. As confirmed by numerical results (see also the plots in Figures 2 and 3), this is still true for $t_l \leq t \leq t_h$. However, also for this model, when the normalized price of anarchy is considered, the two cases are equivalent for $t > t_h$.

Figures 2 and 3 show the behavior of the price of anarchy (as a function of the per-unit parallel trade cost t) for the noncooperative games and global welfare models examined in the paper, for two choices of the set of parameters a, b, and γ. A MATLAB 7.7.0 implementation has been used to generate the two figures. The two figures refer only to the cases of the games modeling parallel trade banning and parallel trade threat, respectively, since $\gamma = 4 > 2$ has been chosen to generate the plots in the first figure, whereas the condition $bt < \frac{3\gamma a}{14b}$ holds only for a small range of values for t in the second figure. In Figure 2(a), which refers to the Bentham global welfare model, the price of anarchy when there is parallel trade threat is always smaller than or equal to the price of anarchy when parallel trade is forbidden. However, both prices of anarchy are close to 1, indicating that the corresponding subgame-perfect Nash equilibria are quite efficient, and no change of rules of the games (due, e.g., to the possible intervention of a policymaker) is really needed to improve their efficiency significantly. A similar situation occurs in Figure 3(a), which also shows that, still for the Bentham global welfare model but for a different choice of γ, there is an interval of values for the per-unit parallel trade cost t for which the price of anarchy when there is parallel trade threat is smaller than the price of anarchy when parallel trade is forbidden, and another interval of values for which the opposite occurs. Differently from the case of the Bentham model, Figures 2(b) and 2(c), which refer to the first specification of the Rawls global welfare model, provide extremely large values for the prices of anarchy when there is parallel trade threat, revealing the need for changing the rules of the game if one is interested to obtain efficient equilibria. In particular, in

Figure 2(b), for sufficiently small values of t (i.e., for $0 \leq t < t_l$), such a price of anarchy is even infinite, according to formula (89). This occurs since in such cases the market B is not served at equilibrium, as shown by formula (71). Instead, such a behavior is not observed in Figure 3(b), as the corresponding threshold t_l is equal to 0, due to the different choice of the parameter γ. As anticipated, Figures 2(b,c) and 3(b) also show that, for the first specification of the Rawls model, the price of anarchy when there is parallel trade threat is always greater than or equal to the price of anarchy when parallel trade is forbidden, and that the equality holds when one considers instead the normalized price of anarchy, and t is sufficiently large. Finally, similar comments can be made for Figures 2(d) and 2(e) and Figure 3(c), which refer to the second specification of the Rawls global welfare model.

Concluding, Table 1 and the numerical results in Figures 2 and 3 show that the results of the comparison in terms of the price of anarchy between the noncooperative games examined in the paper (modeling, respectively, parallel trade banning/parallel trade threat/parallel trade occurrence), are sensitive to the values of the per-unit parallel trade cost and of the relative market size of the two countries, and to the choice of the global welfare function.

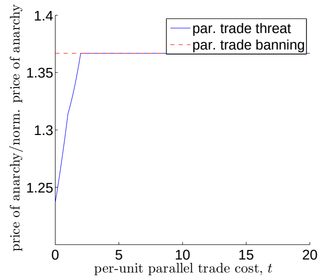
6. Discussion

The price of anarchy is a useful tool to measure the efficiency of equilibrium solutions to noncooperative games. Although originally proposed in [8] for the case of Nash equilibria, it can be extended to other solution concepts (e.g., subgame-perfect Nash equilibria). In practice, when the price of anarchy for a specific noncooperative game is "large", this means that the rules of the game should be changed, in order to obtain much more efficient equilibria. Instead, when it is a "small", no change of rules by a policymaker is really needed to improve the efficiency significantly.

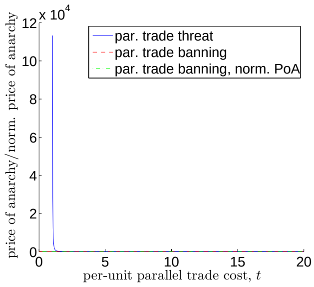
In the noncooperative game-theoretic models of parallel trade banning/freedom examined in this paper, the price of anarchy measures the ratio between the optimal value of the global welfare and its value obtained in correspondence of the "worst" equilibrium. In the paper we have obtained closed-form expressions for the price of anarchy for two noncooperative games proposed in [7] to model the interaction between a manufacturer and a distributor possibly involved in parallel trade of pharmaceuticals, and for a related model proposed in [5]. Three situations have been considered: the case when parallel trade is forbidden, the case when there is parallel trade threat but parallel trade does not actually occur at equilibrium, and the case of actual parallel trade occurrence at equilibrium. Finally,

First game	**parallel trade banning** ($\gamma > 1, C_F = 0$)	
Bentham m.	without normalization:	
	$\frac{16(\gamma^2+1)}{12\gamma^2+7}$	
	with normalization:	
	$\frac{16(\gamma^2+1)}{12\gamma^2+7}$	
Rawls m. (I spec.)	without normalization:	
	$\frac{16}{3}$	
	with normalization:	
	$\frac{8(\gamma^2+1)}{3}$	
Rawls m. (II spec.)	without normalization:	
	8	
	with normalization:	
	$4(\gamma^2+1)$	if $1 < \gamma \leq \sqrt{3}$
	16	if $\gamma > \sqrt{3}$
Second game	**parallel trade threat** ($\gamma > 1, C_F = 0$)	
Bentham m.	with/without normalization:	
	$\frac{\frac{(\gamma^2+1)a^2}{2b}}{\frac{a^2}{72b}(32\gamma^2-4\gamma-1)-\frac{1}{18}(bt^2+at+2\gamma at)}$	if $0 \leq t < t_l$
	$\frac{\frac{(\gamma^2+1)a^2}{2b}}{\frac{a^2}{288b}(124\gamma^2-44\gamma+91)-\frac{1}{36}(4bt^2+2\gamma at-5at)}$	if $t_l \leq t \leq t_h$
	$\frac{16(\gamma^2+1)}{12\gamma^2+7}$	if $t > t_h$
Rawls m. (I spec.)	$+\infty$	if $0 \leq t < t_l$
	$\frac{\frac{(\gamma^2+1)a^2}{4b}}{GW_{PT}^{(R,I,\text{s.p.Nash},i,PTT)}(\gamma,t)}$	if $t_l \leq t \leq t_h$
	$\frac{8(\gamma^2+1)}{3}$	if $t > t_h$
Rawls m. (II spec.)	$\frac{\frac{(\gamma^2+1)a^2}{8b}}{GW_{PT}^{(R,II,\text{s.p.Nash},i,PTT)}(\gamma,t)}$	if $1 < \gamma \leq \sqrt{3}$ and $0 \leq t \leq t_h$
	$4(\gamma^2+1)$	if $1 < \gamma \leq \sqrt{3}$ and $t > t_h$
	$\frac{\frac{a^2}{2b}}{GW_{PT}^{(R,II,\text{s.p.Nash},i,PTT)}(\gamma,t)}$	if $\sqrt{3} < \gamma \leq \frac{5}{2}$ and $0 \leq t \leq t_h$
	16	if $\sqrt{3} < \gamma \leq \frac{5}{2}$ and $t > t_h$
	$+\infty$	if $\gamma > \frac{5}{2}$ and $0 \leq t < t_l$
	$\frac{\frac{a^2}{2b}}{GW_{PT}^{(R,II,\text{s.p.Nash},i,PTT)}(\gamma,t)}$	if $\gamma > \frac{5}{2}$ and $t_l \leq t \leq t_h$
	16	if $\gamma > \frac{5}{2}$ and $t > t_h$
Third game	**parallel trade occurrence** ($1 < \gamma \leq 2, 0 \leq t < \frac{3\gamma a}{14b}, C_F = 0$)	
Bentham m.	$\frac{(\gamma^2+1)a^2}{\frac{11}{13}(\gamma a)^2-\frac{12}{13}(\gamma a)(bt)+\frac{23}{13}(bt)^2+\frac{3}{4}a^2-\frac{1}{13}(\gamma a)a-\frac{4}{13}a(bt)}$	
Rawls m. (I spec.)	$\frac{(\gamma^2+1)a^2}{2\left(\frac{a}{2}-\frac{\gamma a+4bt}{13}\right)^2}$	
Rawls m. (II spec.)	0	

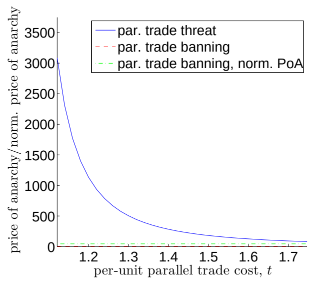
Table 1: Expressions of the price of anarchy obtained for the noncooperative games and global welfare models examined in the paper.



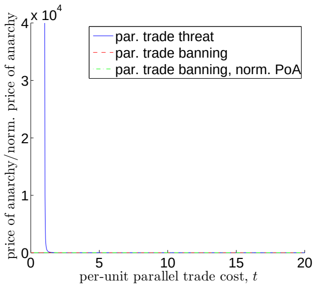
(a) Bentham model



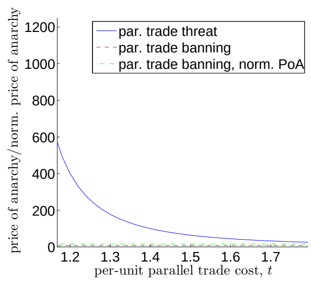
(b) Rawls model (I spec.)



(c) Rawls model (I spec.), zoomed in

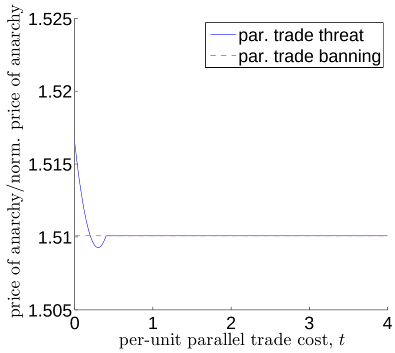


(d) Rawls model (II spec.)

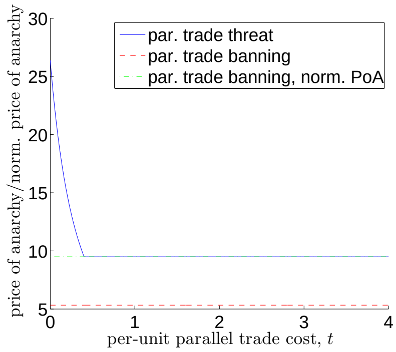


(e) Rawls model (II spec.), zoomed in

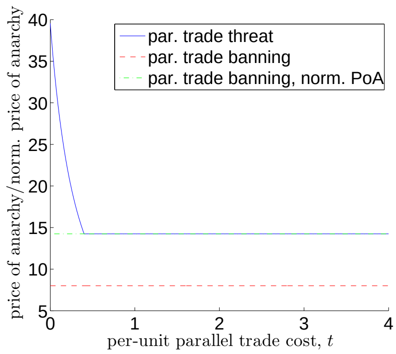
Figure 2: Plots of the prices of anarchy (as functions of the per-unit parallel trade cost t) for some of the noncooperative games and all the global welfare models examined in the paper, for $a = 2$, $b = 1.5$, and $\gamma = 4$.



(a) Bentham model



(b) Rawls model (I spec.)



(c) Rawls model (II spec.)

Figure 3: Plots of the prices of anarchy (as functions of the per-unit parallel trade cost t) for some of the noncooperative games and all the global welfare models examined in the paper, for $a = 2$, $b = 1.5$, and $\gamma = 1.6$.

we have compared the expressions of the price of anarchy obtained for some of the noncooperative games and all the global welfare models examined in the paper. The results of the comparison are sensitive to the values of the per-unit parallel trade cost and of the relative market size of the two countries, and to the choice of the global welfare function. Although the prices of anarchy for the three games have been evaluated in Section 5 under the simplifying assumption of a zero total fixed cost of production (in order to obtain closed-form expressions), the results of the analysis in Section 3 could be used, in principle, to evaluate such prices of anarchy numerically, for the case of a non-zero total fixed cost of production. Up to the authors' knowledge, the application of the concept of price of anarchy to noncooperative games modeling parallel trade of pharmaceuticals is novel. In principle, the evaluation of the price of anarchy and of its normalized version may be extended to other choices of the global welfare function. Moreover, as another possible extension, the price of anarchy could be evaluated also for other noncooperative games modeling parallel trade, such as the ones studied in [2, 4, 6] (e.g., other noncooperative games for which parallel trade actually occurs at equilibrium, when parallel trade is permitted). The results of such a comparison would be useful to measure the efficiency of the proposed solutions, and to detect when policymakers should change the rules of the game in order to increase the value of the global welfare significantly.

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